Author:
Stylianou, Nicos

Title:
On 'Probability'

A Case of Down to Earth Humean Propensities

On ‘Probability’

A Case of Down to Earth Humean Propensities

By

NICOS STYLIANOU

Department of Philosophy
UNIVERSITY OF BRISTOL

A dissertation submitted to the University of Bristol in accordance with the requirements of the degree of DOCTOR OF PHILOSOPHY in the Faculty of Arts.

SEPTEMBER 2020

Word count: sixty-three thousand five hundred twenty

Abstract

Bertrand Russell once said that 'probability' is the most important concept in modern science, especially as nobody has the slightest notion what it means. Little has changed since Russell's pronouncement. Despite the fact that 'probability' appears across the entire spectrum of scientific theories, there does not seem to be even an approximate agreement among philosophers regarding what probability is. Although all the standard interpretations of the concept of probability capture some of the intuitions we assign to the term 'probability', a consensus has been reached in the literature that none provides a satisfactory definition of the term as it appears across our currently best *physical* probabilistic theories. Nonetheless, in order to take seriously what these probabilistic physical theories say about the world, one must be able to tell what the *probabilistic assertions* in these theories mean. That is to say, what makes these *probabilistic assertions* true (or false). The main purpose of this study is to provide an analysis of the concept of probability that allows one to take seriously what *probabilistic assertions* in physical theories say about the world *given* one's commitment that they are an *objective* description of it. The question investigated is thus the following: What could *probabilistic assertions* in physical theories possibly mean given one's commitment to their objectivity?

Dedication and acknowledgements

I dedicate this study to the memory of my sister Stela, the most beautiful person–inside and out–I have ever met and have been fortunate to call my sister. Also, to the memory of my beloved grandmother who passed away earlier this year.

Firstly I would like to thank Dr Catrin Campbell-Moore who supervised me for this study in the last two years. If this study is any good Catrin is to thank, if it is not then it is all my fault. Catrin became my supervisor after Prof. Alexander Bird left University of Bristol. Since then, Catrin was always there to help, to read countless drafts in very short time, always enthusiastic to listen to my–many times nonsensical–ideas and explain to me both what I was doing and what I was doing wrong. Any candidate should consider themselves lucky to have Catrin as their supervisor. Catrin, thanks for everything.

I want to thank Prof. Alexander Bird for the first two years he supervised me in this study and who made clear to me what is–allow the philosophical jargon–metaphysically at stake when one thinks about the concept of probability. I also want to thank my secondary supervisors Prof. Richard Pettigrew and Prof. Samir Okasha for all of their help and support. Also, Prof. James Ladyman for commenting on my draft on 'quantum probabilities', a subject that I am certainly not an expert on.

I want to sincerely thank my examiners Dr Karim Thébault and Dr Luke Fenton-Glynn for the time and effort they put into carefully reading my PhD thesis and for the insightful and constructive comments they provided. Your feedback has been invaluable both for my current and future research. Thanks a lot.

Also, all the people who helped, both via friendly discussions or by reading earlier drafts of this study. As these are too many to name, here are a selected few in alphabetical order: Aadil Kurji, Ana-Maria Creţu, Antonis Antoniou, Geoff Keeling, Guy Lockyer, Lucy James, Nick Axten, Nick Cosstick, Manthos Vathes, Vanessa Seifert. Thank you all.

Of course, my wonderful parents, Anna and Andreas, to whom I basically owe everything. It sounds cliche but in this case it is just the truth. The four of us have always been strong as a family whatever life threw to us.

Last but in no way least, my Daniela. If my study is awarded with a PhD, in my case the letter 'D' stands for her name. Thank you for everything, for your love, your support and for bearing with me throughout these years.

AUTHOR'S DECLARATION

I declare that the work in this dissertation was carried out in accordance with the requirements of the University's Regulations and Code of Practice for Research Degree Programmes and that it has not been submitted for any other academic award. Except where indicated by specific reference in the text, the work is the candidate's own work. Work done in collaboration with, or with the assistance of, others, is indicated as such. Any views expressed in the dissertation are those of the author.

SIGNED: .... DATE: ....

Table of Contents

List of Figures

Figure **Page**

Chapter 1

Introduction

> *"Although we apply the concept of probability in daily life as a matter of course, we find it difficult to say what we mean by the concept 'probable'."*
>
> — Reichenbach (1971, p.3)

Probabilities are used everywhere; meteorologists speak about the probability of rain on a given day, economists talk about the probability of a company going bankrupt or the probability that interest rates are rising. Gamblers deliberate about the probability that the next spin of the roulette will hit a red, that a coin will land heads, a die will roll on an even number and so on. Statisticians talk about the probability of patients developing certain side effects from ingesting a new drug, or the probability that a group of people with certain genes will develop a particular behaviour, for instance to become smokers. Physicists talk about the probability that the particle of an ideal gas will be within a certain sub-region of the system's region as in statistical mechanics, and of the probability of a radioactive atom decaying within a certain time frame as in quantum mechanics.

But what are these probabilities referring to? Do all probabilities allude to the same thing? Do they have the same meaning? Perhaps probability is a pluralistic concept that refers to different entities or quantities depending on the context it is used in. Do the successful applications of probability calculus in the sciences indicate the existence of a distinct entity often called 'chance'? Maybe 'probability' is a reducible term which, as Humeans say, supervenes on the actual history of the world throughout space and time, on the Humean mosaic.

A consensus seems to have been reached in the literature that none of the 'standard' interpretations of probability e.g. the classical, the logical, the subjective, the frequency and the propensity, provide an adequate analysis of 'probability' (Earman and Salmon (1992), Loewer (2001), Szabó (2001b), Hájek et al. (2011), Schwarz (2018)). This consensual conclusion is better formulated as follows: although all these interpretations capture some of the intuitions we usually assign to the term 'probability', none succeeds in giving a satisfactory interpretation of the term as it appears in the propositions of our scientific theories.

A complete examination of the fundamental question *'what is probability?'* is a formidable task. The question is multi-layered, with each layer having implications on how the question is to be approached. The main purpose of this chapter is to specify what part of the question will be addressed in this study and what philosophical perspective will be adopted in this attempt.

To this end, let us begin by separating the question 'what is probability' in two sub-parts. The first, is the *mathematical* question whose concern is to identify the formal features of probability, to identify which is the 'proper' axiomatic theory of probability. The second is the *philosophical* question, where the concern is to identify what sort of 'things' are probabilities (Humphreys et al. (2008), Hájek et al. (2011), Gillies (2012), Bunge (2015)). The philosophical question is usually referred to as the task of providing an *'interpretation of probability'*.

In terms of the mathematical question, on the one hand, there are many *mathematical* theories of probability and there is an ongoing debate over the 'proper' axiomatic theory of probability. On the other hand, Kolmogorov's theory of probability has reached the status of orthodoxy. This study follows this orthodoxy and takes as given that the formal theory of probability is that of Kolmogorov. That is, it is taken as given that Kolmogorov has answered the mathematical question regarding probability. In this thesis the focus is solely on the philosophical question with the mathematical one only being discussed when this is relevant for the philosophical one. In the next section Kolmogorov's mathematical theory of probability is briefly described.

1.1 Mathematical Probabilities

Generally, for the mathematical theory of probability, probabilities are numerical values that a function P assigns to the 'bearers' of probability where the bearers can be either 'events' and/or 'propositions'. That is, the so-called 'universal set' Ω can be either a space of outcomes or space of worlds. In the first case its subsets are called 'events', while in the second case its subsets are called 'propositions'. The mathematical theory of probability remains silent regarding what are the 'proper' bearers of probability.

The initial development of probability theory can be traced back to the mid 17th century and the famous correspondence between Fermat and Pascal.[1] Yet, the axiomatic theory of probability had to wait until 1933 for Kolmogorov to establish the theory of probability in terms of measure theory in pure mathematics.

[1]In its early days probability theory had a purely combinatorial status and could only apply to a finite number of random events. During the 18th century, the theory became entangled with calculus due to Bernulli's 'law of large numbers' and de Moivre's 'central limit theorem'. In brief, the 'law of large numbers' states that an experiment is repeated over a long period of time and as long as the repetitions of the experiment are independent, the frequencies of the outcomes of these experiments converge to an arithmetic mean. De Moivre's 'central limit theorem' states that under certain conditions the normal distribution is an approximation to the binomial distribution.

Kolmogorov's Theory of Probability: Let Ω be a non-empty set (the so called 'universal set') for which $\mathscr{F}$ is an algebra of. Let P be a function from $\mathscr{F}$ to $\mathbb{R}$. P is a 'probability function' and $(\Omega, \mathscr{F}, P)$ a 'probability space' where for every A and B in $\mathscr{F}$, P satisfies the following axioms:

1. (Non negativity): $P(A) \geq 0$

2. (Normalization): $P(\Omega) = 1$

3. (Finite Additivity): $P(A \cup B) = P(A) + P(B)$, if $A \cap B = \varnothing$

When $\mathscr{F}$ is a σ-algebra finite, additivity is extended to 'countable addivity':

$3'$.(Countable addivity): $P\left(\bigcup_{i=1}^{i=\infty} A_i\right) = \sum_{i=1}^{i=\infty} P(A_i)$, where the A_i are mutually disjoint.[2]

Kolmogorov defines the *conditional probability* of A given B by the ratio of their absolute probabilities:

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ given $P(B) > 0$, for every A and B in $\mathscr{F}$. This ratio is undefined if either or both the unconditional (or absolute) probabilities are undefined or if $P(B) = 0$.

1.2 The Philosophical Task of 'Interpreting Probability'

The standard characterisation of the task of providing an 'interpretation of probability theory' is usually formulated as the task of assigning familiar meaning to the primitive terms in the axioms and the theorems of Kolmogorov's mathematical theory of probability with the intention to turn them into true statements regarding some subject of interest (Hájek et al., 2011).

The task needs further clarification. It is not clear what the task of assigning *meaning* to *mathematical* primitives is. That is, it is not clear what it *means* to provide *meaning* to *mathematical* primitives. Consider the following historical remark by Humphreys et al. (2008) which, I believe, shows how one's general philosophy of mathematics affects how the philosophical question–the task of 'interpreting probability'–is to be perceived. It dates back to the 19th century and to Hilbert's 6th open problem in his famous lecture in Paris. Hilbert writes:

> The investigations on the foundations of geometry suggest the problem: *To treat in the same manner, by means of axioms, those physical sciences in which mathematics plays an important part; in the first rank are the theory of probabilities and mechanics.* As to the axioms of the theory of probabilities, it seems to me desirable that their logical investigation should be accompanied by a rigorous and satisfactory development of the method of mean values in mathematical physics, and in particular in the kinetic theory of gases (Hilbert, 1902, p.454) [Emphasis in the original].

[2] σ-algebras are a subset of algebras since all σ-algebras are algebras, but not vice versa. This is because algebras require only that they are closed under pairwise unions whereas σ-algebras must be closed under countably infinite unions.

For Humphreys et al. (2008, p.2) it is clear that for Hilbert at least, probability was viewed as a part of science, not of mathematics. It is beyond our task to examine whether Hilbert was asking for a scientific theory of probability or a mathematical theory of probability. What it is indeed clear is that the aim and accomplishment of Kolmogorov is a probability theory that is part of *pure* mathematics. Kolmogorov (1933, pp.v-1) writes:

> The purpose of this monograph is to give an axiomatic foundation for the theory of probability. The author set himself the task of putting in their natural place, among the general notions of modern mathematics, the basic concepts of probability theory [...] The theory of probability, as a mathematical discipline can and should be developed from axioms in exactly the same way as Geometry and Algebra.

It is thus evident that Kolmogorov's aim and accomplishment is a *purely* mathematical theory of probability not unlike Geometry and Algebra. Consider two of the philosophical perspectives from which one may approach the task of 'interpreting probability', the difference between them ultimately boiling down to one's philosophy of mathematics.

First, one may adopt the perspective of Bradley (2017, p.3) who argues that the (mathematical) probability theory was invented to deal with the pre-theoretical concept of probability such that the "current technical usage of the term is connected to this pre-theoretic usage in the same way that 'force' in physics or 'continuity' in mathematics are connected to folk uses of those terms".

Second, one may adopt the perspective of Gyenis and Rédei (2014, p.3) who argue that measure theoretic probability (understood as $(\Omega, \mathscr{F}, P)$) is not a formal interpretation of the concept probability and must thus not be contrasted with any 'interpretation of probability'.[3] This approach does not deny of course that there are pre-theoretical concepts of probability. It only *denies* that 'mathematical probability', the triplet $(\Omega, \mathscr{F}, P)$, is a formal interpretation of these concepts. In general, the formalist approach rejects the claim that mathematical terms (or primitives) have non-mathematical meaning; neither intuitive nor empirical. Probability theory is thus considered as a purely formal theory, a system of symbols imposing formal constraints on the properties of measures, random variables, and other items in the domain of the mathematical theory. The theory itself has no non-mathematical content.

Without taking a stand on which approach is the 'correct' one, we note that if the former approach is to be adopted, it is perhaps better to follow the suggestion of Hájek et al. (2011) that the task of an 'interpretation of probability' is to be perceived as a Carnapian 'explication'.[4] On the other hand, if the latter approach is to be followed the task of an 'interpretation of probability' is to be conceived as the task of providing an 'analysis' of different concepts of probability.[5]

[3] Gyenis and Rédei (2014) use the notation (X, S, p).

[4] Carnap writes that the task of explication: "[C]onsists in transforming a given more or less inexact concept into an exact one [...] We call the given concept (or the term used for it) the explicandum, and the exact concept proposed to take the place of the first (or the term proposed for it) the explicatum (Carnap, 1962, p.3).

[5] The task of analysis, broadly speaking, is a process of isolating or working back to what is more fundamental by means of which something, initially taken as given, can be explained or reconstructed (Beaney, 2018).

Ultimately, which approach is to be followed in the 'interpretation of probability' cuts down to subtle issues such as one's philosophy of mathematics and other discussions e.g. 'analysis' or 'explication'. The examination of these issues extends well beyond the aim and scope of this thesis. The author's sympathies are with the formalist approach and it is this approach this study follows.[6] Here is how Reichenbach characterises the 'formalist' approach:

> Like all mathematical systems, the calculus of the P-symbol is employed in two conceptions. *In the formal conception we do not give any meaning to the P-symbol, but setup formal relations connecting various forms of expressions.* In other words, we define the P-symbol implicitly by a set of axioms. In the material conception or interpretation we introduce a meaning for the P-symbol in terms of other mathematical or physical concepts (Reichenbach, 1949, p.2) [My emphasis].

Gyenis and Rédei (2014, p.10) note that for the formalist approach one must be careful to distinguish between: (i) pure mathematics, (ii) applications of probability theory, (iii) interpretations of probability, and (iv) our concern at hand, the task of a *philosophical* interpretation of the concept of probability. A brief description of each follows:

(i) Pure Mathematics: As long as the mathematical theory of probability is concerned, 'mathematical probability' is a *purely* mathematical term, it is just the triplet $(\Omega, \mathscr{F}, P)$ and does not come with any intuitive or empirical meaning. That is, the triplet $(\Omega, \mathscr{F}, P)$ is not considered as a formal interpretation of the concept of probability.

(ii) Applications of Probability Theory: Applications of probability is when the triplet $(\Omega, \mathscr{F}, P)$ models phenomena external to mathematics or it is represented in terms of another mathematical structure e.g. as in the case of 'Bertrand Paradox', where probability theory is represented in geometrical terms.[7] We focus on cases where probability theory is applied to phenomena external to mathematics. For instance, an application of probability theory to the non-mathematical phenomenon such as the 'throw of a die' is as follows: Let $\Omega_6 = (1,2,...,6)$ be a six element set of elementary events. $\mathscr{F} = \mathscr{P}(\Omega_6)$ be the set of all subsets of Ω_6; that is, the general events. The probability of throwing i is $P(i) = 1/6$ ($i = 1,2,...,6$). The probability of event A is $P(A) = \sum_i \in_A P(i)$. Thus, the probability of 'throwing an odd number' $P(A) : A = 1,3,5$, is $P(A) = \sum_{i \in (1,3,5)} P(i) = 3/6 = 1/2$.

In this sense, an *application of probability* is simply when *mathematical probability* models phenomena external to mathematics. Gyenis and Rédei (2014) note that an application involves two tasks. The first task is that of *event interpretation* that specifies what the elements in Ω and $\mathscr{F}$ stand for e.g. whether they are the sides of the a die or the possible outcomes of the experiment and so on. The second task is that of *truth interpretation* that clarifies when the

[6]For different positions and a general discussion on philosophy of mathematics see Horsten (2019). For a comprehensive discussion of Carnap's concept of explication see Beaney (2004) and for that of 'analysis' see Beaney (2018).

[7]'Bertrand Paradox' is discussed in Chapter 2 section 1.

proposition '$P(A) = 1/2$' is true or false e.g. because of the symmetry of the die or because of relative frequencies of the six possible outcomes or maybe because of the 'propensity' and so on. In general, a probability measure space is a good *mathematical model* of the targeted phenomenon if it is *descriptively accurate* and *predictively successful*. It is predictively accurate when given the fixed specification of the event and truth interpretations, the proposition '$P(A) = 1/2$' is true about events observed in the past. It is predictively successful when '$P(A) = 1/2$' is true in future observations. These features can be evaluated only by empirical considerations.[8]

(iii) Interpretations of probability: Interpretations of probability are defined as typical classes of *applications of probability theory* that consist of probabilistic propositions that possess some common features. In other words, interpretations of probability are categorisations of probabilistic propositions into classes one derives by the *applications of probability theory* to physical (non-mathematical) phenomena, depending on common features one thinks they share.

(iv) Philosophical Interpretation of 'probability': The task of a *philosophical 'interpretation of probability'* aims to provide an analysis of the common features of each class. In other words, a *philosophical interpretation of probability* aims to provide an *analysis* of the common features that different *interpretations of probability*, different classes of *applications of probability theory*, share. This approach on the task of 'interpretation of probability' aligns with another remark by Hájek et al. (2011) that the task of 'interpretations of probability' would be better called analyses of various concepts of probability and 'interpreting probability' as the task of providing such analyses.

This study takes the 'concepts of probability' and 'interpretations of probability' to refer to collections of different probabilities or more precisely, different *probabilistic assertions* one derives by *applying probability theory* that share common features. That is, each 'interpretation of probability' corresponds to a particular 'concept of probability'. I will be using the term 'concepts of probability' because it is more general and more neutral in terms of connotation.

To conclude, in this study we understand the task of 'interpreting probability' as the task of providing an *analysis* of the common features that different concepts of probability share. Each concept of probability refers to particular *applications of probability theory*. More precisely, it refers to particular probabilistic propositions derived from the *applications of probability theory* to non-mathematical phenomena already categorised into classes–into different interpretations or concepts of probability–depending on common features we consider they share. In a nutshell, this study proposes that the task of a *philosophical interpretation of probability* is to provide an analysis of the common features of different concepts of probability.

This section has discussed how one's philosophy of mathematics, in a way, mandates how the philosophical part of the question is approached. This thesis follows the formalist approach as it

[8]Following Gyenis and Rédei (2014, p.10) we consider that there is nothing peculiar regarding the applications of probability theory, that from this perspective, probabilistic scientific theories are just like any scientific theory. See Humphreys et al. (2008) for a similar claim and for an excellent discussion on how the purely mathematical theory of probability applies to the modelling of non-mathematical phenomena in the empirical sciences.

has been just described.

1.3 Three Concepts of 'Probability'

This section suggests that there are three concepts of probability, one subjective and two objective ones, each of which requires a distinct interpretation, a distinct analysis.

Hacking (2006, p.15) remarks that: "Philosophers seem singularly unable to put asunder the aleatory and the epistemological side of probability". The concept of probability has suffered from equivocation. There is an objective concept of probability–Hacking calls this the 'aleatory side of probability'–where probability of an event is considered a property of the event itself or of its circumstances. Objective probability is a feature of reality, of the world 'out there'. There is also an epistemic or subjective concept of probability where probability is a property of some agent's 'state of mind' or 'degree of belief' regarding whether or not an event will occur e.g. whether 'the sun will rise tomorrow' or that 'the next toss of a particular coin will land heads'.

To be precise, on the one hand we have 'objective probability' which is considered a feature of the external world and no matter what exactly that feature is, as long as it exists it does so irrespective of whether or not there exist (rational) agents with 'subjective probabilities'. Objective probability is 'out there'. On the other hand, 'subjective probability' is about the 'mental dynamics' or 'mental states' of (rational) agents. Therefore, interpretations of 'objective probability' and 'subjective probability' are not competing interpretations of the same concepts. Rather, they are distinct concepts each of which refers to different sorts of phenomena with each requiring distinct interpretation.

Ismael (2011) notes that *'objective probability'* suffers its own equivocation. She writes:

> One of the most persistent sources of confusion in discussions of probability is the failure to distinguish general from single-case probability. General probabilities apply to classes of events and the basic form is conditional. The general probability of B among A's is written $Pr(A|B)$. Single-case probabilities, by contrast, to *particular* events rather than classes [...] and the basic form is *un*conditional (Ismael, 2011, p.418) [Emphasis in the original].

This study follows Ismael and presupposes that 'general probabilities' have the following common features: (1) they apply to *classes of events* or *event types* and (2) they always come with a reference class and because of that their basic form is conditional. For example, the probability of *a* toss of *a* fair die to 'land on an even number' always comes with a reference class e.g. tosses of a fair die. A reference class is indispensable of 'general probabilities'; without it the concept of 'general probability' is not well-defined. On the contrary, we consider 'single-case objective probabilities' to share the following common features: (1) they apply to particulars, to token events, (2) their basic form is unconditional or absolute and as such they do not come with an explicit reference class. For instance, the probability of *this* particular die thrown at *this* particular time, tossed by that person, in such and such a way, lands on an even number.

Ismael also notes three positions one may hold regarding the metaphysical relationship between 'general probability' and 'singe-case probability':

> One may hold that: (i) they are both primitive forms of probability, neither reducible to the other, (ii) general probabilities are definable in terms of single-case probabilities, or (iii) single-case probabilities are definable in terms of general probabilities (Ismael, 2011, p.419).

At least for now we adopt the first position; that they are both primitive concepts that are not reducible to each other, nor definable in terms of each other and as such, they require distinct interpretations, distinct analyses. There are two reasons for adopting the first metaphysical position regarding the relation between 'general probability' and 'single-case probability':

First, the features we use to distinguish the two concepts of objective probability are diametrically opposite: 'general probability' always comes with a reference class and its basic form is conditional whereas 'single-case probability' or 'chance' has no explicit reference class and its basic form is unconditional or absolute.

Second, and most important, if we suppose that 'general probability' is to be understood as frequencies of event *types* such as the 'sequential tosses of a coin', the 'sequential spins of a roulette' etc., then it is impossible to define 'single-case objective probability' or 'chance' in terms of general probabilities without committing the fallacy of division; that is, without inferring that because probability is a property of a sequence, of a reference class, it is also a property of a particular, of a token, of that sequence or reference class. On the other hand, the issue of whether 'general probability' can be defined in terms of 'single-case probability' is a subtle one. To determine if this can be done, one needs to first interpret 'single-case probability'. This is one of the central aims of this thesis; I undertake this task in Chapter 5.

For these reasons, this study takes 'general probability' and 'single-case probability' as different concepts of objective probability–as different classes of *applications of probability theory*, of probabilistic propositions–sharing these common features. That is, 'general probability' and 'single-case probability' are considered as distinct objective concepts of probability that each requires distinct analysis. In this sense, I follow the suggestion of Ismael (2011) that 'standard' interpretations such as frequency and long-run propensity are to be considered competing interpretations of the concept of objective *general probability* while single-case propensity interpretations and Humean reductions are to be considered competing interpretations of the concept of *single-case probability*.

To summarise, two claims are being made. *First*, that the concept of probability suffers from 'tri-vocality': the term 'probability' may refer either to the subjective concept of probability or to one of the objective concepts of 'general probability' and 'single-case probability'. 'Single-case probability' is often called 'chance'.[9] *Second*, that the task of 'interpreting probability' is considered as the task of providing an analysis of each concept of probability.

1.4 The Central Question

This sections aims to clarify what the task of offering an interpretation of the objective concepts of probability entails. This will require specifying what approach to scientific theories is assumed for examining the central question, as well as setting two further 'working hypothesis'. The aim is to refine the central question of the thesis and in turn make the task of this study more explicit.

The author has sympathies to the operationalist/conventionalist approach in scientific theories as this has been articulated in the works of Mach (1900), Poincaré (2003), and Reichenbach (1965) among others. I will not argue for this position. It is a well-advocated and well-criticised position which, similarly with any other approach to scientific theories, has its virtues as well as its drawbacks.[10] Hereinafter I provide a brief description of the main idea of the operationalism/conventionalism approach in scientific theories.

Central to the operationalist/conventionalist approach is the condition that the concepts of mathematical science must be introduced by *explicit definitions*. This led to an extensive discussion on the importance of the *principle of coordination* between formal theories and the physical reality.[11] The establishment of coordination principles is considered a fundamental issue for the proponents of the operationalist/conventionalist view in scientific theories whose sympathies typically lie with the formalist view on mathematics. According to the formalist view on mathematics, mathematics is solely concerned with abstract relational structures, denying any connection between mathematics and sensory experience. Thus, establishing a connection between abstract mathematical theories in physics and the concrete physical phenomena they intend to represent becomes a core problem usually called the 'problem of coordination'.

This problem comes into sight in at least two cases: (1) when one attempts to provide a general coordination between the theory and the physical reality and (2) when one attempts to provide an operational definition of a given theoretical term in terms of measurable quantities. Reichenbach (1965) was amongst the first who attempted to solve the aforementioned cases where the problem of coordination appears.

Regarding the first case, Reichenbach (1965) posits that there is a special class of non-empirical physical principles he calls coordinative principles, principles of coordination or axioms of coordination, whose function is to establish the connection between mathematical structures and non-mathematical physical phenomena in such a way that the mathematical manipulation of the targeted phenomena can be achieved. These principles are to be viewed as preconditions

[9] I use the terms 'single-case probability' and 'chance' interchangeably.

[10] See Gillies (1972) for a critique of the operationalist/conventionalist approach in scientific theories.

[11] See Padovani (2017) for a comprehensive discussion on the–often reflected–importance of the principle of coordination.

for establishing empirical knowledge. As a consequence, they themselves cannot be similarly; that is, empirically, established.

Friedman (2004, p.37) characterises these principles as necessary presuppositions for deriving a theory's empirical claims. By necessary presuppositions he means that the acceptance of these principles is a prerequisite for the empirical claims of a given theory to acquire non-mathematical meaning. He illustrates his claim by giving a detailed exposition of the role these principles play in Newtonian physics and in Einstein's relativistic physics. In brief, he notes that in the context of Newtonian physics, the concept of absolute acceleration invoked in Newton's laws of universal gravitation does not have non-mathematical meaning unless there is a frame of reference where Newton's laws of motion hold. That is, the only way one can give non-mathematical meaning to Newton's law of universal gravitation is by presupposing that there is a frame of reference (an 'inertial frame') where Newton's laws of notion hold. Without presupposing that, the question of the truth or falsity of Newton's law of universal gravitation does not even arise. In this sense, Friedman (2004, p.77) concludes that the laws of motion in Newtonian physics have the status of coordination principles. They function as the general rules for coordinating the abstract mathematical structure of Newtonian physics–i.e. infinite Euclidean space, uniformly traversed lines, straight lines in Euclidean space, temporal intervals where the state of motion traverses equal spatial intervals–and the sensory experience they intend to describe i.e. the observable relative motion of objects in the solar system.

Friedman (2004) observes that over the transition from Newtonian physics to Einstein's relativistic physics and his general theory of relativity (henceforth GR) about space, time and motion, these principles have been replaced by new ones that essentially play the same role as those in Newtonian physics. More precisely, over the transition from Newtonian physics to GR, the infinite Euclidean space has given its place to a four-dimensional semi-Riemannian manifold of a variable curvature endowed with the Lorentzian transformation metric in such a way that the 'light cone' is characterised at each point in the manifold, infinitesimally imitating the flat 4-d Minkowski spacetime geometry of special relativity. In addition, the inertial trajectories of Newtonian physics gave their place to the 4-d geodecis of the semi-Riemannian metric as representations of the state of natural motion. Lastly, Newton's law of universal gravitation gave its place to Einstein's field equations that govern that spacetime metric in such a way that they relate the latter to the 'stress-energy tensor', the new mathematical representation of matter.

In the case of GR, the coordination is established by two Einsteinian principles: (1) the light principle (light is always propagated in empty space with a constant velocity which is independent of the state of motion of the emitted body) and (2) the principle of equivalence (the motion of free-falling 'test particles' in a gravitational field is always a geodesic motion). The light principle coordinates physical phenomena with the Loretzian 4-d spacetime metric and the principle of equivalence completes the coordination. These principles are fundamental presuppositions of the workings of physical reality. Without these, the properly empirical laws of general theory of relativity–i.e. Maxwell's equations of electromagnetism in a relativistic sense and Einstein's field equations of the electromagnetic field–do not have non-mathematical meaning; they remain parts of pure mathematics. It is in this context that these principles are to be viewed as presuppositions of physical reality whose acceptance ensures the correspondence between the abstract mathematical structures and the non-mathematical phenomena these intend to represent.

Once the correspondence is established, the physical claims of the theory can be empirically tested i.e. one can use Einstein's field equations to calculate the advancements in the perihelion of Mercury. Yet, such experimental procedures do not provide empirical testing or support of the mathematical part of the theory, nor of the principles of coordination. For instance, the four-dimensional semi-Riemannian manifold of a variable curvature is just a purely mathematical description of certain mathematical structures whose axioms and theorems are justified purely mathematically. The experimental procedures do not provide empirical evidence in favour of the coordinative principles either; such procedures could not have been set up unless the coordinative principles were not already in place i.e. without accepting the principle of equivalence, Einstein's field equations remain a purely abstract mathematical description of the abstract mathematical objects of semi-Riemannian manifolds. These procedures test the empirical laws formulated in terms of mathematical structure under consideration. Yet, the mathematical formulation of the laws only becomes possible if the principles of coordination are already accepted. That is, they provide empirical evidence for Einstein's field equations given that the principle of equivalence is already accepted. Otherwise, the evidence could not have been generated at all. In other words, the acceptance of these non-empirical physical principles as truths by convention is a prerequisite for a certain theory to acquire non-mathematical content.

The principles of coordination discussed thus far *do not* provide operational definitions of the elements of the mathematical theory they coordinate with physical reality. Rather, they establish ideal cases where actual physical phenomena can approximate in the limit. For instance, the light principle holds exactly true only in strictly infinitesimal regions and similarly, the principle of equivalence holds exactly true only for infinitesimal 'test particles'; only the motion of truly infinitesimal 'test particles' in a gravitational field is exactly a geodesic motion (Friedman, 2004, p.79).

The problem of coordination also appears when one attempts to provide an operational definition coordinating an abstract theoretical concept with an actual physical phenomenon. Reichenbach (1965) summarises this by considering that a scientific theory is a set of theoretical/mathematical terms denoting abstract relations between these abstract terms that acquires empirical substantiation *if and only if* these abstract terms are connected with some physical quantities through *measurement*. Otherwise, a theory remains an abstract structure without any empirical meaning; that is, without *measurement* we do not have a physical theory but rather a theory of pure mathematics. The connection between the abstract/mathematical terms and physical quantities can be established by specifying a principle of coordination that enables the quantification and consequently the mathematical manipulation of the targeted physical quantities. To establish a principle of coordination, one must first identify: (1) its domain and range, and (2) the kind of relation that makes the coordination between the *mathematical* terms and the *physical quantities* possible. If the physical quantities are not already defined, any attempt to establish a principle of coordination will be circular.

Consider the following two examples from the history of science where the problem of coordination is discussed, indicating that: (1) principles of coordination are indispensable for the regulation of the use of these quantity-terms, (2) any attempt to establish such a principle ends up circular and (3) in order to avoid the circularity, a convention is required.

'Length': Reichenbach, in his discussion regarding the coordination of 'length' by defining its metric unit (meter) with the circumference of the earth divided by 40 million, notes that in order to know 'the circumference of the earth' a principle of coordination that connects 'length' to the physical reality must have already been established. However, this is what the principle of coordination 'A meter is the circumference of the earth divided by 40 million' is supposed to establish (Reichenbach, 1927). When discussing the alternative principle for the coordination of 'length' that 'a measuring rod retains its length when transported' he also notes that this principle that regulates the term 'length' has to be assigned with a definition-like status (Reichenbach, 1927). The reason is the following: In order to examine whether the statement 'a measuring rod retains its length when transported' is true, a principle of coordination for 'length' must have already been established. Otherwise, we would end up in a circularity as it is possible that a universal and experimentally undetectable force exists that equally distorts every object's length when transported . Consequently, Reichenbach concludes that the only way of determining whether the 'length of the rod remains the same before and after its transportation' is by having the metric of length already established. *The conventionalist 'solution'* to the problem of coordination claims that the circularity of coordination can be avoided by assigning a definition-like status to the *ultimately conventional* principle of coordination. That is, to avoid the circularity of the coordination of 'length', we need to take the statement 'a measuring rod retains its length when transported' as expressing an *arbitrary* rule regulating the concept of equality of length . A convention is necessary as the principle of coordination claims something about the empirical realm whose truth-value cannot be examined before its acceptance. It is its very acceptance that enables the theoretical concept 'length' to acquire physical (non-mathematical) meaning.[12]

'Temperature': Given that principles of coordination are mere conventions, then for every established principle an alternative–and in some respect equally good–principle may exist. In other words, our choice of a certain principle of coordination over its perhaps equally good alternatives is undetermined by the nature of things. Mach (1900) illustrates this problem with the following example: The principle of coordination for quantifying 'temperature' is: '$t =$

[12]For further discussion on Reichenbach analysis of 'length' see Dieks (2010) and Van Fraassen (2010).

$f(v)$' where v is the substance used in the thermometer. At the time of the construction of the thermometer, there were more than one, equally good, candidates for v. Except for Galileo's volume, Lambert's pressure of the mass of gas was also an available candidate for v. Mach argues that the coordination of 'temperature' with physical reality through the principles of coordination '$t = f(\text{volume})$' rather than '$t = f(\text{pressure of the mass of gas})$' was a mere convention. Choosing one principle of coordination over the possible alternatives may have non-trivial consequences. That is, even if there is a relation between the scales derived from the two candidates for v, the relationship is an entirely contingent feature of physical reality–it might be the case that two equally good candidates result to non-related scales. In counterfactual terms, if an alternative statement were accepted as a principle of coordination, it would not necessarily lead to measurement outcomes that are somehow related to those resulted by the accepted principle.[13]

In all the aforementioned cases a principle of coordination is required in order to establish the correspondence of these theoretical terms with *measurable* physical quantities; that is, to *operationally* define these terms. In all of these cases the establishment of the principle of coordination ultimately requires convention. Yet, it is of particular importance for the task of this study to note that operationalism/conventionalism is *compatible* with scientific objectivity (or scientific realism). Tal (2017) writes:

> Scientific theories and models are commonly expressed in terms of quantitative relations among parameters, bearing names such as 'length' [...] An operationalist or conventionalist would argue that the way such quantity-terms apply to concrete particulars depends on nontrivial choices [...] [regarding] the way the relevant quantity is measured. *Note that under this broad construal, realism is compatible with operationalism and conventionalism.* That is, it is conceivable that choices of measurement method regulate the use of a quantity-term and that, given the correct choice, this term succeeds in referring to a mind-independent property or relation [My emphasis].

We emphasise the fact that operationalism/conventionalism in scientific theories is compatible with scientific objectivity, where 'scientific objectivity' stands for the claim that our currently best scientific theories provide a mind-independent and identical to all observers description of physical reality. That is, they describe how the world 'out there' *is* and not what one (rational or not) thinks about the world. This point is essential for the two working hypotheses adopted throughout this study.[14] They are as follows:

Working Hypothesis 1: Our currently best scientific theories describe the world objectively, irrespective of what anyone thinks about the world. That is, we suppose that probabilistic assertions, in physics in particular, are to be interpreted in an objective manner rather than a subjective one. That is, when it comes to physical theories we consider that their probabilistic

[13]For further discussion of Mach's analysis of 'temperature' see Van Fraassen (2010).

[14]Tal (2017) uses the term scientific realism. I prefer the term 'scientific objectivity' instead because 'scientific realism' is an extremely philosophically loaded term the discussion of which is not the aim of this thesis.

statements are to be interpreted in an objective manner either as 'general probabilities' or 'single-case probabilities'. For this reason we do not examine attempts that aim for a subjective interpretation of the probabilistic assertions in physics. It is important to stress out that by 'objective' I do not assert that our currently best scientific theories are *the* true descriptions of the world but rather, that they are amongst the possible ways the world can be objectively–mind independently and identical to all observers–described. This point requires further elaboration that takes place in Chapters 4, 5 and 6. For now, consider the following simple analogy as an illustration of this point. All of the following statements *objectively* describe the *real* event 'team X won the football game 3-0': (i) 'team X did not lose', (ii) 'team X won', (iii) 'team X won and scored more than one goal' and (iv) 'team X won with a 3-0 score' in the following sense: even if in descriptions (i), (ii) and (iii) the lack of complete information of the 'states of affairs' corresponding to the event under consideration is evident, these are just as objective descriptions of the event as description (iv) is. That is, even when proposition (iv) is known, the truth value of the rest of the propositions remains unaltered. The fact that one comes to know proposition (iv), does not make any of the other ones false.

Working Hypothesis 2: We consider statistical mechanics and quantum mechanics as our currently best probabilistic physical theories as a consequence of their immense predictive and descriptive success. That is, we consider that the probabilistic assertions of both statistical and quantum mechanics describe the world 'out there' because they pass the strictest of standards for a theory to count as descriptively and predictively successful. Roughly, both statistical and quantum mechanics make probabilistic assertions. The descriptive and predictive success of both is mainly evaluated based on the probabilistic predictions they make. That is, the actual frequencies produced by the repeating quantum experiments and experiments in statistical mechanics have been found to converge to the values predicted by the probability *postulates* or *conventional rules/principles* of each. In this minimal sense, allow me to say that these theories have been experimentally 'confirmed', 'verified' or 'empirically corroborated' because they predict the experimental findings; they predict the value that the frequencies of outcomes tend to converge.

We consider the 'probability postulate' or the 'Born rule' as the empirically significant probabilistic assertion of quantum mechanics coordinating the quantum formalism with real physical systems. The Born rule: consider that a given quantity O has an associated operator $\widehat{O}$ written as $\widehat{O} = \sum_i o_i \widehat{\Pi}(i)$ where o_i are the distinct values of the operator and $\widehat{\Pi}(i)$ projects into the subspace of states with eigenvalue o_i. In case that quantity O is measured on a quantum system with state $|\psi\rangle$, then: (i) the only possible outcomes of that measurement are the eingenvalues o_i of the operator and, (ii) the probability that the measurement results to o_i is $\mathrm{P}(O = o_i) = \langle\psi|\widehat{P}(i)|\psi\rangle$. In rough terms, the Born rule enables the calculation of the frequencies of various outcomes when a certain type of measurement is performed. Of course, what is the physical meaning of the 'wave function' and of 'measurement' is a very subtle matter.[15] Either way, the experimental (empirical) meaning of 'quantum states' is always given in terms of possible outcomes and their actual relative frequencies.[16]

Statistical mechanics targets isolated physical systems consisting of a large number of particles. We consider as the empirically significant probabilistic assertions of statistical mechanics the following: the 'probability' postulate of (Boltzmann's) statistical mechanics asserts that the 'probability that the state of the system lies in a sub-region S of region Γ' is the ratio '$\mu(S)$ / $\mu(\Gamma)$', where $\mu(S)$ is a measure of the volume associated with the space S and $\mu(\Gamma)$ is a measure of the volume associated with the space Γ, usually a Lebesgue measure. That is, the probabilistic assertion of statistical mechanics is that: '$P(S) = \mu(S) / \mu(\Gamma)$'.[17]

With that said, the main focus of this study is the conceptual problem of the objective concepts of probability and this extends well beyond statistical and quantum mechanics. It will be a problem for any future physical theory that makes *probabilistic assertions*. Miller (2015, p.195) expresses this conceptual problem fittingly: "One of the principal challenges confronting any objectivist theory of scientific knowledge is to provide a satisfactory understanding of physical probabilities". This is indeed the case. Insofar as physical theories make probabilistic assertions, then an objective reading of these scientific theories requires an objective reading of their probabilistic assertions.[18]

We are now in a position to state the problem and the central question investigated in this study.

The Problem: Given the consensus that none of the 'standard' interpretations of probability provide an adequate analysis of 'probability' as the term appears in our scientific assertions, how can the teachings of our currently best scientific theories–which supposedly describe how the world is–'be taken seriously', if we do not know what their probabilistic statements mean? I borrow the expression be 'taken seriously' from Schwarz (2018) as I believe it precisely captures the essence of the problem. That is, it seems reasonable to except that for one to take something seriously one needs to know what that something means.[19] The central question of this study is thus the following:

> *What could probabilistic assertions in physical theories possibly mean given one's commitment to their objectivity?*

[15] For a discussion on the measurement problem in quantum mechanics see Leggett (2005) and Myrvold (2016a).

[16] 'Quantum probabilities' are discussed in Chapter 6.

[17] We discuss probabilistic assertion is statistical mechanics in Chapter 5.

[18] What will be argued in this thesis, I believe, can extend to any physical theory that earns the title of 'descriptively accurate and predicatively successful'. This of course will depend on one's criteria for a theory to count as such. We won't examine this issue any further.

[19] 'Meaning' is a subtle issue but entering such a debate extends well beyond the scope and aim of this thesis. We follow the standard approach that a statement is meaningful insofar as we can specify its truth conditions; insofar as we can provide an analysis of what makes the statement true (or false). For a comprehensive discussion around theories of meaning see Speaks (2010).

As Miller (2015) notes, an understanding of objective probability is a central challenge of an objectivist theory of scientific knowledge. In particular, the analyses of the objective concepts of probability–'general probability' and 'single-case probability'–should be able to tell us what the probabilistic assertions of statistical and quantum mechanics say about the world *given* the supposition that they are objective descriptions of the world and *given* that we specify which is the relevant concept of objective probability in each case.

In this thesis we take as given that the probabilistic assertions of our current best probabilistic physical theories like statistical mechanics and quantum mechanics describe the world 'out there' and given that supposition, our aim is to make sense of what their *objective* probabilistic assertions could possibly mean. Thus, in case that our attempt is successful it would, indirectly, make a case for the possibility of an objectivist theory of scientific knowledge. That is, scientific theories, particularly in physics, describe how the world *is* and not what anyone thinks about the world. For the aforesaid reasons, this study focuses on the objective concepts of probability, and mainly on the concept of 'single-case probability'. It examines how in physics in particular, 'single-case probability' is intimately tied to one's metaphysical convictions such as one's reading of modality *de re* and (or) one's stand on determinism (or not).

1.5 A Summary of What Follows

Chapter 2 provides a critical overview of some of the 'standard' interpretations of probability categorised in terms of the relevant concepts of probability: either *'subjective probability'*, *'general probability'*, or *'single-case probability'* also called 'chance'. *Section 2.1* discusses the Classical, the Logical and the Subjective interpretations of the concept of probability that have been grouped together because one way or another, an epistemic or subjective concept of subjective probability is involved. It suggests that only the Subjective interpretations are tenable. *Section 2.2* discusses interpretations of the concept of 'general probability' such as long-run frequency (Venn (1888)), infinite frequency (von Mises (1964), Reichenbach (1949)) and long-run propensity interpretations (Popper (1959), Gillies (2012)). It suggests that long-run actual frequency interpretation provides the most tenable interpretation of the *concept* of general probability. *Section 2.3* focuses on single-case propensity interpretations of the concept of single-case probability. It distinguishes between two sorts of single-case propensity interpretations, 'Hard propensity' and 'Hybrid propensity' interpretations. Hard propensity interpretations are considered those exclusively concerned with 'single-case probability' such as the interpretations of Popper (1990), Miller (1995) and Giere (1973b), whereas Hybrid propensity interpretations of 'single-case probability' are considered those that make essential reference to the concept of 'general probability'; the focus is on the interpretations of Mellor (2004) and Suárez (2013). It is argued that none of the single-case propensity interpretations discussed provides a coherent analysis of the concept of single-case probability. Regarding Hard propensity interpretations, it is argued that their main problem is that they introduce a separate entity/property–the propensity–to interpret the concept of single-case probability, but they do not inform on what propensity is. Hybrid propensity interpretations, it is argued, are threatened with incoherency.

Chapter 3 notes that usually the interpretation of 'single-case probability' as (irreducible) propensity comes in conjunction with a commitment to indeterminism (Giere (1973b), Miller (2015), Suárez (2013)). Propensities are thus considered features of an indeterministic world. The chapter suggests that single-case propensity interpretations that associate the concept of single-case probability with that of indeterminism first require a coherent analysis of 'indeterminism'. Accordingly, it discusses the concept of indeterminism by Belnap and Green (1994, p.1), the central idea of which is that "at a given moment in the history of the world there are a variety of ways in which affairs might carry on". It considers two ways for spelling out the distinction between the actual and the possible an analysis of the concept of indeterminism requires. First, the Classical Possibilism reading of modality *de re* and its distinction between *is* and *being* and its 'revised' version between *actual existence* and *possible existence*. Second, the temporal rather than modal distinction between actuality and possibility in terms of 'real possibilities' by Müller et al. (2018). It argues that when actuality is distinguished from possibility in terms of ontic modalities à la classical possibilism, one faces 'Quine's challenge' that an ontology that includes ontic modalities is incoherent. On the other hand, the temporal concept of 'real possibilities' makes essential reference to the negation of determinism and as such it does not suffice for a positive characterisation of the concept of indeterminism. Its principal aim is to motivate the Humean Propensity theory proposed in Chapter 5 that disentangles 'single-case probability' from 'indeterminism' and, more generally, from irreducible modalities.

Chapter 4 takes into account the suggestion of Belnap and Green (1994) that any concept of 'single-case probability' must rely on that of 'possibility' and it discusses Humean ontologies that manage to avoid these problems by, in one way or another, reducing the concept of possibility to an overarching actuality, paving the way for the remainder of this study. More specifically, it describes the metaphysical doctrine of 'Humean Supervenience' and the 'Humean Project' distinguishing between *'Down to Earth Humeanism'* and *'Possible Worlds Humeanism'*. 'Down to Earth Humeanism' stands for the metaphysical doctrine of Humean Supervenience and a Strict Actualism regarding of modality *de re*. Second, 'Possible Worlds Humeanism' stands for the metaphysical doctrine of Humean Supervenience and 'Modal Realism' reading of modality *de re* by Lewis (1986b).

Chapter 5 proposes an interpretation of 'single-case probability' or 'chance' in terms of Humean propensities. This interpretation is nothing more than a complementary reading of Lewis' Best System Analysis of 'chance' and Szabó's 'no chance' interpretation (Lewis (1980), Lewis (1994), Szabó (2001b), Szabó (2007b), Szabó (2010)). I call this interpretation of the concept of single-case probability the 'Humean Propensity' because, I claim, it captures the features of the hard propensity interpretations while avoiding their problems and without deviating from the metaphysical doctrine of Humean Supervenience. Most importantly, it does not commit to the deterministic (or not) nature of physical reality. The proposed interpretation is, I believe, compatible with both Down to Earth Humeanism and Possible Worlds Humeanism.

Chapter 6 reflects on the common view that quantum phenomena in Bell/Aspect experiments cannot be understood within the framework of the classical-relativistic (or pre-quantum) 'world view', with this causing trouble for the metaphysics of Humean Supervenience (Maudlin (2007b), Karakostas (2009)). It focuses on the assumption of measurement independence in Bell's theorem noting that it has metaphysical implications. Sufficient conditions for measurement independence require a metaphysical commitment to irreducible modalities. Nevertheless, it is argued that in the 'classical relativistic world-view' examined in Bell's theorem there are no *sufficient* conditions for measurement independence. Two types of arguments in favour of measurement independence are discussed: (1) arguments that experimentalists have 'free will', understood as the ability to do (i.e. measure) otherwise; (2) arguments against a physical cosmic 'conspiracy'. It suggests that the super-deterministic reading of Bell/Aspect experiments that bites the cosmic 'conspiracy' bullet should not be easily neglected, especially upon purely philosophical grounds. 'Conspiratory' or not, it allows one to consider that quantum probabilistic assertions are about *real/physical* relative frequencies of measurement outcomes, where the conditioning events are *real/physical* events of 'choosing' a measurement setting to measure a certain observable. In this manner, the EPR correlations and the Bell/Aspect results can be coherently–both conceptually and technically–interpreted as *objective* long-run relative frequencies of measurement outcomes relative to different measurement settings. Thus, the chapter concludes that, at least as far as Bell/Aspect results are concerned, Humeans can provide–a consistent with the classical-relativistic 'world view'–interpretation of quantum probabilities as long-run frequencies. In terms of the distinction between two objective concepts of probability—general probabilities and chances-–it suggests that quantum probabilistic assertions are to be categorised as general probabilities instead of chances.

Chapter 2

'Standard' Interpretations

"Problems in the foundations of probability bear at least indirectly, and sometimes directly, upon central scientific, social scientific, and philosophical concerns. The interpretation of probability is one of the most important such foundational problems."

— Hájek et al. (2011)

The chapter provides a critical overview of the standard interpretations of 'probability'. It builds upon the distinction between the three concepts of probability discussed in Chapter 1–'subjective', 'general' and 'single-case probability'–and considers, among the standard interpretations, as *competing* those that are after the same concept of probability. In other words, it does not consider that all of the standard interpretations are *competing* interpretations of the *same* 'probability'.

Section 1 briefly discusses the Classical, the Logical and the Subjective interpretations of 'probability' which attach, one way or another, an epistemic or subjective element to the concept. The focus then is on the interpretations of the concepts of objective probability that are relevant for this study: 'general probability' and 'single-case probability'. *Section 2* discusses interpretations of 'general probability' such as the Long-Run Frequency, the Infinite Frequency and the Long-Run Propensity interpretations. *Section 3* discusses interpretations of 'single-case probability', such as the single-case propensity interpretations of Popper (1990), Miller (1995), Giere (1973b), Mellor (2004) and Suárez (2013).

The main aim of this chapter is twofold. *First,* it argues that the long-run frequency interpretation provides the most tenable interpretation of the concept of 'general probability'. *Second*, it argues that while some single-case propensity interpretations–e.g. by Popper (1990), Miller (1995) and Giere (1973b)–capture important features of 'single-case probability', they ultimately fall short in providing a coherent analysis of the concept of single-case probability; they do not reveal what propensity *is*. This paves the way for the remainder of the thesis, one of the main focuses of which is to determine how to interpret 'single-case probabilities'.

2.1 Classical, Logical and Subjective Interpretations

This section discusses the Classical, the Logical and the Subjective interpretations of probability. It concludes that among the three only the subjective interpretation is a tenable interpretation of 'probability'. In nutshell, it is argued that the main problem of the Classical interpretation is the lack of justification for the epistemic neutrality in the absence of any evidence the Principle of Indifference entails. Regarding the Logical interpretation, it is noted that Ramsey's objection to its claim that there are logical relations of probability between pairs of propositions remains unanswered. Lastly, it suggests that the subjective interpretations of probability as degrees of belief analysed operationally in terms of betting quotients provides an analysis of the concept of 'subjective probability'. Nevertheless, it results to many problems that one who seeks an operationally respectable concept of subjective probability may need to bite.

2.1.1 The Classical Interpretation

The Classical interpretation dates back to Laplace (1825) who defines the probability of an outcome as the ratio of favourable cases to the number of equally possible cases. Outcomes are considered to be 'equally possible' when there is no reason to prefer one over the other. 'Equally possible' outcomes have received a more precise formulation by Keynes whose Principle of Indifference states:

> [I]f there are no known reasons for predicating of our subjects one rather than another of several alternatives, then relative to such knowledge the assertions of each of these alternatives have an equal probability (Keynes, 1921, p.24).

To put it briefly, the classical interpretation considers that, for cases where the Principle of Indifference applies, all possible outcomes should be assigned the same probability. Thus, the probability of an event is just the fraction of the total number of possibilities in which the event occurs. This entails that the classical interpretation asserts that probabilities can be determined based on a priori considerations, just by examining the 'space of possibilities'. Consider what the classical interpretation says about the 'toss of a coin': Suppose that we are about to toss a coin and there are two possible states of affairs the toss may lead to: the coin either lands heads or it lands tails. In situations like this where we are unable to predict the outcome of an event with certainty, Laplace says we can use the calculus of probability to predict the outcomes. We can do so "partly because of our ignorance and partly because of our knowledge" (Laplace, 1825, p.6). It is partly to our ignorance since, at least for Laplace, if complete information about the laws of nature and the initial conditions of the universe were known we would have been able to predict the outcome of the coin toss with certainty.[1] For Laplace, it is partly to one's knowledge in

[1] This is due to Laplace's commitment to determinism. As his demon hypothesis states: "Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of

the sense that one knows something about the physical situation at hand that enables them to conclude that the two possible outcomes of the toss are either 'heads' or 'tails', and somehow, to exclude the possibility that the coin will explode. What is important to note is that the classical interpretation supposes that one can exclude the possibility that the coin will explode based on *prior* considerations. Suppose further that one believes that heads will occur as much as they believe tails will occur. In such situations, the classical interpretation maintains that 'heads' and 'tails' have an equal probability, half each.

The classical interpretation assigns probabilities to outcomes only in cases where the Principle of Indifference applies. The Principle of Indifference applies to two cases. First, when symmetrically balanced evidence is available. Second, when no evidence is available. Both are cases of having no evidence for preferring one outcome over another and thus are cases where the principle of indifference should be applicable. Call the former cases as *type 1* cases and the latter as *type 2* cases. Thus, the classical interpretation assigns probabilities only in *type 1* and *type 2* cases.

For *type 1 cases*–when we have symmetrically balanced evidence for each outcome–Hájek et al. (2011) note that the classical interpretation becomes circular: in order to characterise some evidence as symmetrically balanced, some sort of weighting of the evidence for each outcome is in order. It is unclear whether this can be done without essential reference to probability-theoretic terms. For instance, it would only lead to circularities if symmetrically balanced evidence was characterised in terms of the equality of their conditional probabilities.[2]

On the other hand, the classical interpretation can avoid the circularity but then it becomes a frequency interpretation in disguise. Consider the following example by Szabó (2001b): Suppose a symmetric die with six faces numbered from 1 to 6. When the die is thrown in the usual way there are six possible outcomes this may lead to. Thus, there are six possible outcomes (1, 2, 3, 4, 5, 6) three of which (1, 3, 5) are considered favourable. Since there is no evidence for preferring one outcome over the alternatives, the classical interpretation suggests that each outcome has equal probability. Thus, according to the classical interpretation the probability of getting an odd number equals half.

Suppose further that at the moment of the throw of the die the history of the universe is–at least epistemically–branching into six branches and the possible histories of the universe are categorised into six classes where each class corresponds to one of the outcomes 1 to 6. Three out of six branches correspond to the event 'the die lands on an odd number' such that the probability 'the die lands on an odd number' equals 3/6. Yet, this will still be the case even if the die was biased; the number of possible outcomes as well as the number of favourable outcomes would

the beings who compose it —an intelligence sufficiently vast to submit these data to analysis— it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes" (Laplace, 1825, p.4).

[2]For instance, given evidence E and possible outcomes $A_1, A_2, .., A_n$ the evidence E is symmetrically balanced just in case that $P(A_1|E) = P(A_2|E) = ... = P(A_n|E)$. Yet, such a characterisation of symmetrically balanced evidence makes essential reference to probability-theoretic terms.

have been the same. One could argue that in the latter case the Principle of Indifference cannot be applied because the die is not any more symmetric. Yet, it is not symmetric *in terms* of its mass density. Nevertheless, even a standard die is not *completely* symmetric in respect to *all* of its properties e.g. different numbers are written on the different faces of the die etc. This is simply to say that if the die was *completely* symmetric then one would not have been able to distinguish between the six outcomes each time a die was thrown.

On the other hand, for one to consider that the asymmetry of the different numbers written on each face of the die is irrelevant for 'probability', one is forced to draw a distinction between relevant and irrelevant asymmetries where the relevant asymmetries have to be those that have an impact on the 'probability' of the six possible outcomes. Nevertheless, the concept of probability that justifies the distinction between relevant and irrelevant asymmetries is some sort of frequentism. That is, when one says that the relevant asymmetries are those that have an impact on the probabilities of the six possible outcomes, the 'probability' seems to refer to the observable fact that the biased die–the die that is asymmetric in terms of mass density–produces one outcome *more often* than the other. In cases like this, the classical interpretation makes essential reference to the early frequency conception of 'probability' as frequency of Aristotle who, in his Rhetorics, says 'the probable is that which *happens often*'.

The principle of Indifference applies also to *type 2* cases; that is, in cases where we have no evidence whatsoever. For example, the first coin ever tossed, the first die ever thrown, the first roulette ever spun etc. Suppose that one can somehow identify the possible outcomes of an experiment before ever performing the experiment. That is, suppose that it is somehow known based on a prori considerations that the coin will land either heads or tails and it won't explode even if one is about to toss the first coin ever tossed. It is unclear what is the justification for assigning equal probability to each outcome rather than assigning probability 0.1 to outcome heads and 0.9 to tails or 0.7 to heads and 0.3 to tails etc. In situations like this, one has no prior experience of how coin tossing works. If in the absence of any evidence one considers that all possible cases are equally probable then it is like receiving information from no information. That is, one receives information about the situation at hand from the fact that one has no information whatsoever about the situation at hand (Hájek et al., 2011). In general, the epistemic neutrality in the absence of any evidence that the Principle of Indifference entails cannot be justified.

A common objection against the coherency of the classical interpretation is considered to be Bertrand's Paradox. Bertrand's Paradox intends to convey that the Principle of Indifference that the classical interpretation relies upon leads to inconsistencies. It can be applied in different ways to the *same* situation such that different probabilities are assigned to *same* event. This violates the axiom of probability theory that every event has a unique probability. Bertrand's (1889) famous formulation of the paradox asks the probability that a randomly picked chord (R) of a circle will have greater length than the side of an equilateral triangle edged in that circle. *Choosing* three different parameters as the randomly picked chords and applying the Principle of Indifference to each–since there is no evidence for preferring one of the picked chords over the others–one ends up with three different probabilities for the event in question; that is, 'a randomly picked chord (R) of a circle has greater length than the side of an equilateral triangle edged in that circle'. Consider also the formulation of the paradox by Van Fraassen (1989):

Suppose a facility that produces cubes whose side-length is between [0, 1] meter and consider the question 'what is the probability that a randomly chosen cube has side-length between [0, 1/2] ?'. If we suppose that the production process is uniformly distributed over side-length, the answer, according to the classical interpretation, is 1/2. There is no evidence to favour the possibility that the side-length of the randomly chosen cube is somewhere in the interval [0, 1/2] over the possibility that it is somewhere in the interval [1/2, 1] or vice versa. By employing the Principle of Indifference one assigns equal probability to all possible cases. Thus, the favourable case e.g. cubes with side-length [0, 1/2], over all possible cases e.g. cubes with side-length [0, 1], equals 1/2.

Suppose further that the following is an *equivalent restatement* of the previous question: A facility produces cubes with face-area [0, 1] meters and consider the question 'what is the probability that a randomly chosen cube has face-area between [0, 1/4] meters?'. Supposing that the production process is uniformly distributed over face-area, the answer is 1/4. There is no evidence to favour the possibility that the face-area of the randomly chosen cube is somewhere in the interval [0, 1/4] over the possibility that it is somewhere in the interval [1/4, 1/2] over the possibility that it is somewhere in interval [1/2, 3/4] over the possibility that it is somewhere in interval between [3/4, 1]. By the Principle of Indifference we assign equal probability to all possible cases. Thus, the probability 'a randomly chosen cube has face-area between [0, 1/4] meters' is 1/4.

Suppose that the following is also an *equivalent restatement* of the previous question: A facility produces cubes with volume between [0, 1] cubic meters and consider the question 'what is the probability that a randomly chosen cube has volume between [0, 1/8] cubic meters?'. Supposing that the production process is uniformly distributed over volume, the answer is 1/8. As in the previous cases, there is no evidence to favour the possibility that it is somewhere in the interval [0, 1/8] over the possibility that somewhere in the interval [1/8, 2/8] and so on. By the Principle of Indifference, we assign equal probabilities to all possible cases. Thus, the probability 'a randomly chosen cube has volume between [0, 1/8] cubic meters' is the number of the favourable cases e.g. cubes with volume between [0, 1/8] over all possible cases e.g. [0, 1/8], [1/8, 2/8],...,[7/8, 1], which equals 1/8.

Bertrand-type cases intend to show that the *same event* e.g. 'a randomly picked chord of a circle has greater length than the side of an equilateral triangle edged in that circle' or 'the side-length of the cube is somewhere in the interval between [0, 1/2]', can be assigned with different probabilities just by reformulating the question. Thus, it contradicts the axiom of probability theory that every event has a unique probability.

On the other hand, Gyenis and Rédei (2014) have argued that despite that Bertrand's paradox feels like a paradox, in reality it is not. They note that the *mathematical* concept of the 'Principle of Indifference' is a coherent interpretation of the mathematical theory of probability. They analyse Bertrand's Paradox in terms of measure theoretic concepts showing that the Principle of Indifference–as a mathematical concept–can be consistently formulated for probability spaces with an infinite number of random events whose probability is given by a Haar measure.[3] The so-called 'paradox' points to a *provable mathematical fact* that:

> [O]ur freedom to choose measure theoretic isomorphic probability theories to describe the same random phenomenon manifests the conventionality of naming random events in probabilistic modelling (Gyenis and Rédei, 2014, p. 2).

That is, the way the question is formulated *matters* for probability measures of the classical interpretation. In effect, when one formulates the question in a *different* way, one asks the probability of a *different* event. Consider Reichenbach (1949, p.2) who write that in "the material conception or interpretation we introduce a meaning for the P-symbol in terms of other mathematical or physical concepts".[4] In Reichenbach's terminology, Gyenis and Rédei (2014) point out that 'Bertrand's Paradox' is an admissible interpretation of Kolmogorov's *mathematical* theory of probability in terms of another *mathematical* structure; in the particular case in *geometrical* terms. The so-called 'paradox' is a provable mathematical fact that there is some kind of freedom in constructing such a representation.

This however is not to be seen as an argument for the feasibility of the classical interpretation of 'probability' due to the previously mentioned well-known problems that arise when one tries to link the classical interpretation to phenomena external to mathematics. That is, in *type 1* cases–when one has symmetrically balanced evidence for each outcome–the Principle of Indifference renders the classical interpretation either circular or a frequency interpretation is disguise. In *type 2* cases–when one has no evidence about the situation whatsoever–the epistemic neutrality in the absence of any evidence that the Principle of Indifference entails lacks justification.[5]

2.1.2 The Logical Interpretation

The Logical Interpretation asserts that if all *rational* agents had the same evidence available regarding a hypothesis or proposition, then they should have the same 'degrees of belief' or 'credence' in that hypothesis or proposition. In effect, the logical interpretation defines 'probability' as a 'degree of partial entailment' and in turn, 'the degree of partial entailment' as 'rational degree of belief' (Gillies, 2012, p.31). The logical interpretation can be understood as a theory of 'partial entailment' where partial entailment is thought to be a 'natural' generalisation of full

[3] A Haar measure is referring to any locally compact Hausdorff topological group which has a unique non-zero left invariant measure that is finite on compact sets.

[4] See discussion in Sec 1.2.

[5] For a detailed discussion and a defence for the Principle of Indifference in terms of the Principle of Maximum Entropy type of inference see Uffink (1995).

entailment in deductive logic. Consider the following example: Suppose premise E stating that so far in all cases where water has been boiled it did so at 100 Celsius. Also, consider hypothesis H stating that the next time that water will be boiled, it will do so at 100 Celsius. The logical interpretation says that despite that Hume–with his problem of induction–taught us that H does not logically follow from E, that is, E does not 'fully' entail H yet, E 'partially' entails H because E provides evidence for H.

Ramsey notes that this implies that for the logical interpretation there are logical relations of probability between pairs of propositions which can be perceived or comprehended. He raises the following objection: "Take the simplest possible pairs of propositions such that 'This is red', and 'That is blue' or 'This is red' and 'That is red' whose logical relation should surely be easiest to see, no one, I think, pretends to be sure what is the probability relation which connects them (Ramsey, 1926, p.162). I certainly do not.[6] If one shares such an inability then the logical interpretation will not be considered as a tenable interpretation of 'probability'.[7]

2.1.3 The Subjective Interpretation

The subjective–also called personalist–interpretation defines probability with an agent's 'degrees of belief' to a particular proposition, for example 'it will rain tomorrow in Bristol', 'the sun will rise tomorrow' and so on. But, what does it mean for an agent to have particular 'degrees of belief'? Or more generally, what *are* degrees of belief? The task of providing an analysis of the concept of degrees of belief is to answer this question. The different accounts that aim to do so typically fall under three broad categories: *operationalist* accounts that define 'degrees of belief' in terms of the operations the concept is measured by, *functionalist* accounts that analyse the concept in terms of preferences and more broadly in terms of its role in decision making, and *primitivist* accounts that consider degrees of belief as a primitive concept that requires no further analysis. Since this study adopts the operationalist/conventionalist framework in philosophy of science that requires the concepts of mathematical science to be introduced by *explicit definitions*, the natural account to adopt is the operationalist analysis of the concept of degrees of belief. Our discussion will be limited to this.[8]

[6] See discussion on the Principle of Indifference in sec. 2.1.1.

[7] Gillies (2012) notes that this is the main reason that led many to conclude that the logical interpretation is not a tenable interpretation of 'probability'. For a detailed discussion on the logical interpretation see Gillies (2012, pp.25-49), for a defence see Franklin (2001) and for an overview see Hájek et al. (2011).

[8] Here is a very brief description of the alternative approaches to operationalism. In short, functionalist accounts typically have the form of a representation theorem: some axioms regarding qualitative preference are stated and then it is derived that an agent who obeys these axioms can be represented in terms of a utility and a probability function. Her preferences are represented by *the maximising expected utility* calculated by these two functions (for different representation theorems see, amongst others, Ramsey (1926), Savage (1972), Lewis (1974), Maher (1993), Cozic and Hill (2015) and for a comprehensive overview Zynda (2017) and Eriksson and Hájek (2007)). The functionalist element of this analysis is highlighted by Eriksson and Hájek (2007, p.196) who regard that the representation theorem by Ramsey (1926) defines degrees of belief as "whatever fills the *role* of being multiplied by utilities in the expected utility representation". Elliott (2017, p.378) generalises this claim suggesting that the different analyses of the concept of degrees of belief in terms of representation theorems are functionalist in spirit. That is, one way

Operationalist accounts aim to reduce the concept of degrees of belief in terms of actual or hypothetical betting quotients. The *actual betting analysis* along the lines of De Finetti roughly defines the concept in the following manner: an agent's degrees of belief in A is p *if and only if* p units of utility is the price at which the agent would buy or sell a bet that pays 1 unit of utility if A, and 0 units if not A, given the presuppositions that for any A there is only one such price usually called an agent's 'fair price for the bet on A' and utility is linear. The *hypothetical betting analysis* on the other hand does not require *actual* betting behaviour but rather, betting quotients the agent *would* accept. For instance, Howson and Urbach (2006) propose that degrees of belief are defined as the betting quotients one regards as fair no matter if she takes such a bet or not; they are defined as the betting quotients one considers to offer no advantage nor disadvantage to either side of a hypothetical bet. The general idea is that at the price the agent considers fair, she would be indifferent between accepting either side of the bet.

Let me briefly describe some of the *many* problems that the betting interpretation comes with, as they seem to be bullets that a proponent of operationalism–and more specifically of the betting analysis of the concept of degrees of belief–may need to bite.[9] Recall that according to the betting analysis, an agent's degrees of belief in A is p *if and only if* p units of utility is the price at which the agent would buy or sell a bet that pays 1 unit of utility if A, and 0 units if not A, given the presuppositions that for any A there is only one such price and that utility is linear. This definition indicates that the betting analysis of degrees of belief *postulates* a strict conceptual relation between degrees of belief and actual betting behaviour.

The main issue with this postulate is that there seem to be many intuitive reasons why betting quotients may not correspond to degrees of belief. For example, an agent may have beliefs that lack manifestation in betting behaviour or cases where the agent has reasons to bet in a manner that does not correspond to her actual degrees of belief. Regarding the former, one could imagine a completely apathetic agent who despite having opinions, she lacks corresponding desires that typically stimulate one to bet. For instance, it may be possible to conceive apathetic agents–maybe some sort of Buddhist monks–who lack behavioural manifestation of their mental states all together (Hájek et al., 2011). Also, one may have reasons to misrepresent her true degrees of belief when making a relevant bet so she can potentially exploit an incoherency in someone else's betting prices (Hajek and Hitchcock, 2016). Or, maybe in the case that one threatens to shoot an agent unless she accepts a certain bet then it seems that the agent has very

or another, representation theorems link the concept of degrees of belief to the role it plays in an agent's effort to maximise her expected utility. On the other hand, primitivist accounts deny that an analysis of the concept of degrees of belief is required. Rather, the general idea is that it is a primitive concept that need not and should not be analysed any further. In effect, according to primitivism "the project of analysing degrees of belief was misguided from the start" (Eriksson and Hájek, 2007, p.204). One of the reasons that primitivism appears to be an attractive position is the numerous problems that both the operationalist and the functionalist accounts come with (for a more detailed discussion on these problems see Eriksson and Hájek (2007, pp.186-204)). Notwithstanding, it is questionable if rejecting the task of providing an account of the concept of degrees of belief can be a satisfactory response, at least for one who deems the task as important.

[9] For a comprehensive overview of the problems of the betting analysis see Eriksson and Hájek (2007, pp.185-190) and Zynda (2017) and for an overview see Hájek et al. (2011).

good reasons to accept the bet even if the payoff rate of the bet has nothing to do with her actual degrees of belief (Hájek et al., 2011). In these cases the betting interpretation mischaracterises the agent's actual degrees of belief.

Moreover, it is unclear how one could separate an agent's epistemic attitude to the proposition under consideration from their attitudes towards the act of betting itself. For example, two agents who may have identical opinions towards a proposition may accept very different odds for betting on that proposition depending on their attitude towards the act of betting; the savvy gambler may accept less favourable odds than the risk-averse agent. In such cases their betting behaviour does not reflect their actual degrees of belief towards the propositions they are betting on.

Also, as Ramsey (1926) observes, the act of betting itself may alter the state of affairs in the world and consequently one's opinion regarding these states of affairs. For example, consider a tennis player who bets a large amount of utility units on the proposition 'I will win the next game' and, by doing so, her stress level becomes overwhelming, such that her opinion in regard to the proposition at hand has been altered just by the act of placing the bet.

There are also certain propositions that one may have opinion about but betting on them would not pay out at any point the payoff would matter. For instance, consider the proposition 'there is no life after death'. A bet on this proposition could not be settled, not even in principle, at least in this lifetime. It is counter-intuitive to conclude that one cannot have degrees of belief in such a proposition. Yet, this seems to be the conclusion one derives by the betting analysis of degrees of belief.

A possible way an operationalist may deal with some of these problems is to follow a hypothetical betting analysis. This account avoids the problems arising from defining degrees of belief in terms of *actual* betting behaviour. Yet, the proposal is susceptible to the problems that arise by the essential reference the hypothetical betting analysis makes to the notion of betting e.g. as aforementioned, having beliefs about a proposition but no incentive to place a bet since it could not pay out, not even in principle, at a point it would matter ('there is no life after death'). Also, as Eriksson and Hájek (2007, p.189) note, hypothetical betting analysis is essentially modal; degrees of belief are no longer defined in terms of actual betting quotients but in terms of the betting quotients the agent *would* accept. The modal element that enters the analysis does not fit well with the empiricist/operationalist inclinations that motivate the betting analysis of the concept of degrees of belief to begin with.

Summing up this brief discussion, on the one hand, at least, the actual betting analysis provides an operationally respectable conception of degrees of belief. That is, it takes at par value the operationalist remarks of De Finetti (1972, p.76) that "in order to give an effective meaning to a notion and not merely an appearance of such in a metaphysical-verbalistic sense an operational definition is required" and of Ramsey (1926, p.165) that a "degree of a belief is just like a time interval; it has no precise meaning unless we specify more exactly how it is to be measured". On the other hand, this analysis comes with a variety of problems. Now, if we consider that the main aim of an operationalist is to provide an operationally respectable conception of degrees of belief, the problems of the betting analysis are bullets that she may be willing to bite. At the end of the day, the operationalist is aware of the restrictiveness of her dogma that the concepts of mathematical science are to be introduced by *explicit definitions* and that operational definitions typically clash with many of the intuitions we usually assign to the concept under examination. It is her–often constructive–scepticism regarding arguments from intuitions in general that led her to adopt the operationalist dogma to begin with.[10]

For the sake of the argument, let us accept the operationalist betting analysis of degrees of belief as given. We may also want to examine whether such an analysis of degrees of belief is an *admissible* interpretation of probability theory; that is, whether degrees of belief obey the axioms of probability theory, the 'Probabilism thesis'. The most prominent argument that employs the betting analysis to show that *rational* degrees of belief should obey the axioms of probability (up to finite additivity) is the Dutch book argument first developed by Ramsey (1926) and De Finetti (1972). In brief, a Ramsey/De Finetti Dutch book argument for Probabilism is a series of bets that are bought and sold at prices that no matter what the outcome of the bet turns out to be, they collectively result in a certain loss. The argument presupposes that degrees of belief are defined in terms of betting quotients and proves that if they violate the axioms of probability, the agent who upholds them can be Dutch-booked. The idea is that one who accepts a bet that has no possibility of winning is irrational. The overall conclusion of the argument is that rationality requires for one's degrees of belief to obey the probability calculus. When this is the case, her degrees of belief are said to be coherent.[11]

Given the betting analysis of degrees of belief, the Dutch book argument provides *pragmatic* grounds for considering that degrees of belief of a rational agent should obey the calculus of probability. This, in turn, provides an argument for Probabilism; it demonstrates that *rational degrees of belief are probabilities*.

To conclude, this section has briefly discussed the Classical, the Logical and the Subjective interpretations of probability, suggesting that only the subjective interpretation is tenable. More specifically, given the operationalist framework this study adopts, the analysis of 'subjective probability' as degrees of belief measured in terms of betting quotients provides an operationally

[10]For a discussion of how the operationalist/conventionalist typically treats concepts such as length and temperature see Chapter 1, pp.12-13.

[11]Dutch book arguments for Probabilism are not exclusive to the betting analysis of degrees of belief. In general, they are *pragmatic* arguments for rationality whose structure of reasoning is roughly the following: one assumes that degrees of belief are related in a specific way to one's decisions such that an agent with such degrees of belief should make certain choices when certain situations arise. Then, the argument shows that if an agent's degrees of belief violate the relation assumed to hold between degrees of belief and decisions, her decisions would have unsatisfactory consequences. From this, the argument concludes that an agent whose degrees of belief violate the relation between degrees of belief and decisions under consideration is irrational (Pettigrew, 2020, p.1). For a comprehensive discussion on Dutch book arguments for probabilism and for further constraints that some subjectivists accept and others reject, as well as for a new proposal on developing Dutch book arguments in terms of epistemically possible worlds see Pettigrew (2020, pp.8-13, 21-14).

meaningful concept of degrees of belief. The standard Dutch-book argument for Probabilism provides *pragmatic* grounds for considering that *rational* degrees of belief should obey the axioms of probability theory. On the other hand, the operational character of the betting analysis of degrees of belief comes with the usual problems of operationalism, some of which have been discussed. Yet, considering that the main aim of the operationalists is to provide an operationally respectable conception of degrees of belief, they may be willing to accept the problems that come with it.[12]

2.2 'General Probability'

This section discusses interpretations of the concept of 'general probability' such as long-run frequency, infinite frequency and long-run propensity interpretations.[13] It suggests, amongst the available options, that the long-run frequency interpretation of the concept of general probability is the most plausible.

2.2.1 Frequentism

Frequency Interpretations of probability were pioneered in the works of Venn (1888), von Mises (1964) and Reichenbach (1971) amongst others. According to frequency interpretations, probabilities are objective features of the world associated with *collections of events*. That is, for the frequency interpretation 'probability' is not a concept that can be assigned to an individual experiment, to *token* events, but only to a long sequence of repeated events, experiments etc. There are two versions of frequency interpretations of probability; *Actual Frequency interpretations* (or Long-Run Frequentism) and *Infinite Frequency Interpretations* (or Hypothetical Frequentism).

2.2.1.1 Actual Frequentism

Actual frequency interpretation defines probability of an event as the relative frequency the event occurs in a finitely long sequence. A probability space is identified by specifying a countable finite ensemble of elementary events and the probability of an event equals the relative frequency of the event in the finite ensemble. When an actual frequentist says that 'the probability of heads is half' she says that the relative frequency of outcome 'heads' in the particular sequence is half (e.g. in a finite ensemble of 500 coin tosses, for 250 times the outcome has been heads). In general, an actual frequentist defines the probability 'a coin landing heads' as follows: she tosses a coin for many times and records the outcome of each toss, writes down the number of times that the outcome heads appears in the sequence and divides that by the number of tosses. Also, an actual

[12]These interpretations will not be discussed any further. The focus of this study is on the *objective* concepts of probability.

[13]Chapter 1 assigns the following common features to the referents of 'general probability': (1) they are features of external reality, (2) they always come with a reference class and as such, and (3) their basic form is conditional.

frequentist would say that without an ensemble or reference class of actual repetitions of coin tosses, the probability 'heads' is not a well-defined concept; for actual frequentism, probabilities are *always* conditioned to some well-defined reference class or ensemble.

In general, for long-run frequency interpretation, probability refers to objective features of the world; probabilities exist in the world of experience and they can be accessed by calculating the corresponding relative frequencies. It is very simple, both technically and conceptually, and provides an operationally meaningful definition of probability in terms of actual frequencies. However, this comes at a price. It has limited applicability since there are probability spaces and measures that cannot be interpreted in terms of actual frequentism. It also comes with the general problems of operationalism.

2.2.1.2 Probelms of actual frequentism

Some of the main problems of the actual frequency interpretation follow and where possible, responses are suggested (for a comprehensive discussion see Hájek (1997)):

Firstly, due to its operationalist nature, actual frequentism considers–perhaps contra to our intuitions–that a coin that is never tossed does not have a probability of heads (or tails) because it does not generate any actual outcomes. Similarly, counter-intuitive is the fact that according to actual frequentism, a coin that is tossed only once has a probability of tails (or heads) of either 0 or 1 no matter if it is biased or not.

In addition, actual frequentism cannot meaningfully assign non-trivial (with values other than o or 1) probability to single-cases. Probability is assigned to the entire ensemble of a repeated event. For instance, 'p(heads) = 1/2' may say in one hundred tossed coins, fifty have landed heads. Yet, it does not say anything about the probability of a single experiment of the ensemble. It is meaningless to assign a probability with value other than 1 or 0 to a single instantiation of an experiment e.g. to the 73rd coin-toss, since the favourable outcome 'heads' will either occur or it will not. The problem is that there are many events that can be considered unrepeatable but we still find it intuitive to think that non-trivial probabilities can be attached to them i.e. the 2019 election in the UK, the 2020 world-cup final, certain events in the early history of the universe etc. Actual frequentism can only assign probability 1 or 0 to such events. Perhaps actual frequentists could follow von Mises (1964) and insist that the problem here is with our intuitions and go on to consider that the concept of single-case probability is nonsense. Or, if one accepts the distinction between the concepts of single-case probability and general probability, she could say that long-run frequentism solely aims to interpret the concept of general probability.

Furthermore, the size of the finite ensemble (N) that actual frequentism defines probability by is arbitrary chosen. Yet, probability is always conditioned and heavily depends on the size of N. For instance, in the case of p(heads), if N is an odd number, p(heads) cannot be 1/2. If N is an even number, then p(heads) might be 1/2. To put this problem in different terms, if a fair coin is defined as one that has p(heads) = 1/2 and p(tails) = 1/2, then according to actual frequentism, tossing the coin 101 times or for any odd number of times, it becomes a priory truth that the coin is biased since it is impossible for p(heads) (or p(tails)) to be 1/2 (Hájek, 1997, p.82). This is a consequence of the fact that for actual frequentism, probability is a property of the sequence rather than a property of the coin-toss, the repetitions of which resulted to that sequences.

Also, actual frequentism excludes genuine probabilistic propositions. For example, probabilistic propositions composed by irrational numbers i.e. p(A) = = $1/\sqrt{2}$, cannot be interpreted in terms of finite frequencies. This is a serious drawback, since such probabilistic propositions appear in our scientific theories i.e. quantum mechanics. A possible response that actual frequentists may give is to bite the bullet and deny that there exist irrational probabilities. At the end of the day, no experiment could ever reveal their existence.[14]

Moreover, actual frequentism assigns probability solely to the finite ensemble from which the relative frequency has been calculated. It says nothing regarding the probability of the same event described by a different ensemble. For instance, the relative frequency of heads in the ensemble E_1 = h, h, h, t, t, h, h, t, h, h is 7/10. Thus, p(heads)= 7/10 . This probability has nothing to say regarding p(heads) as described by the ensemble E_2 = h, t, t, h, h, t, t, t, t, t, which resulted from tossing a different coin or even the same coin but at a later time. The relative frequency of heads in E_2 is 3/10. Again, this is a direct consequence of the fact that actual frequentism considers probability as a property of a specific sequence. When the sequence changes, the probability changes as well.

Lastly, actual frequentism faces the reference-class problem, which can be described as follows: an event E has a probability p relative to a reference class C but the same E has a different p relative to reference class C'. However, none of the reference classes stands out as the right one. Reichenbach (1949, p.374) describes the core of this problem fittingly: "If we are asked to find the probability holding or an individual future event, we must first incorporate the case in a suitable reference class. An individual thing or event may be incorporated in many reference classes, from which different probabilities will result". Albeit this problem appears only if we consider actual frequentism applicable to single cases, it is often portrayed as a distinct objection. Let's assume, for the sake of argument, that actual frequentists can somehow manage to assign meaningful probabilities to single cases. Then, the reference class problem indicates that the probability of the single case depends heavily on the class we assigned the single case to. Each case can belong to an infinite number of reference classes. There is also no indication that there is a unique class that can be considered privileged. Thus, there is no upright answer to the proper ensemble of events based on which the probability of the single case should be calculated; there is no single answer for what the probability of a single case should be.[15]

All in all, on the one hand finite frequentism has the advantages of reducing probability to well-understood features of the world i.e. relative frequencies, and the conception of probability as such is utilised in actual scientific practice. On the other hand, it also clashes with many

[14]Chapter 6 suggests an interpretation of quantum probabilities along these lines.

[15]Hájek (2007) argues that the reference problem is a problem for all standard interpretations of probability.

pre-theoretical intuitions one may have about probability. At the end of the day, any philosophical theory comes with its problems and in the particular case the severity of most of these problems depends on one's pre-theoretical intuitions. The clash of actual frequentism with our intuitions is mostly due to its inability to handle single-cases; that is, the problem of the single-case and to that extend that of the reference-class. I suggest that these problems can be avoided by maintaining that actual frequentism is solely concerned with the concept of general probability and by considering general probability as a fundamentally different concept than that of single-case probability. Given the distinction, it should not be expected for long-run frequentism to handle single-cases to begin with. Notwithstanding, the rest of the problems are bullets that an actual frequentist seems to be forced to bite.

2.2.1.3 Limiting Frequentism

Infinite Frequency Interpretation defines probability with the limiting frequency with which an outcome occurs in a series of similar events when the series continues to infinity. More precisely, it identifies a probability space by specifying a countably infinite ensemble of elementary events where the probability of an event equals the relative frequency of that event in the countably infinite ensemble. The relative frequency in an infinite ensemble is *by definition* the limit of the relative frequency in the finite initial segment of the ensemble.[16] The infinite frequency interpretation solves some of the problems of the long run-frequency interpretation, especially since it can interpret a broader spectrum of probability spaces. Yet, when considered as an analysis of the concept of general probability it faces serious challenges (these will be examined shortly).

The focus is on von Mises (1964) concept of probability as limiting frequency. As a justification of this consider a reference to the authority of Kolmogorov (1998, p.387) who writes that: "the basis for the applicability of the results of the mathematical theory of probability to real 'random phenomena' must depend on some form of the *frequency concept of probability*, the unavoidable nature of which has been established by von Mises in a spirited manner". Kolmogorov distinguishes between von Mises' *mathematical* theory of probability and his interpretation of the concept of probability as frequency; he considers that the applicability of his own mathematical theory of probability must depend on von Mises' *concept* of probability. In general, for the formalist approach on the task of 'interpretation of probability' this study adopts von Mises' concept of probability is not to be restricted to his *mathematical* theory of probability. The latter is not considered a formal interpretation of the former. The focus in von Mises' *concept* of probability as frequency and his mathematical theory is discussed only when it is relevant for his concept of frequency.[17]

[16]Martin-Löf (1966) provides a proof that for a large class of probabilities, the limit of their relative frequencies does exist.

[17]For how the *mathematical* theory of probability of von Mises–his two mathematical axioms (discussed shortly)–can be interpreted in terms of the *mathematical* theory of Kolmogorov's see Gillies (2012, pp.109-112,150-168).

2.2.1.4 Von Mises limiting frequentism

In von Mises' frequentism, probabilities are associated with *collections of events* or *mass phenomena*. Gillies (2012), in an insightful remark notes that the examples that von Mises cites as instances of repetitive events and mass phenomena are divided into three categories. *First*, examples of 'games of chance' where the subject matter is a long sequence of throws of a particular die, a long sequence of tosses of a particular coin, a long sequence of spins of a roulette, and so on. *Second*, examples of 'biological statistics' where the subject matter may be the set of English women who were 70 years old in 2017 or the set of certain plants grown in a certain field. *Third*, examples from physics such as the molecules of a particular sample of gas. In all his examples, a particular 'attribute' occurs at each event that constitutes the set of repetitive events or mass phenomena, with such attribute varying from one event to another. Accordingly, each repetitive event or mass phenomenon is associated with certain attributes. For instance, on each toss of the coin either 'heads' or 'tails' occurs, each of the English women either dies before reaching the age of 71 or she survives her 71st birthday, the plants in a particular field either produce certain amount of fruits or they do not and each of the molecules of the gas have a certain velocity or they do not. These attributes constitute what von Mises calls an 'attribute space'.

A 'collective' is according to von Mises (1964, p.12) "a sequence of uniform events or processes which differ by certain observable attribute say colours, numbers, or anything else". A collective can be of two sorts (Gillies, 2012). It can be a *mathematical collective* that consists of an infinite sequence $e_1, e_2, ..., e_n, ...$ such that for all n, $e_n \in \Omega$. Or, it can be an *empirical collective* that actually exists in the world 'out there' and can be observed. For instance, an empirical collective can be a sequence of spins of a roulette that took place on a specific day at a specific time and at a specific location etc. An empirical collective is restricted of course to finite number of members–no roulette can be spun for an infinite number of times. In the mathematical theory of von Mises, an empirical collective is represented as a mathematical collective.

Von Mises' *mathematical collective* obeys two axioms which he considers to reflect empirical laws established through direct observation generalised by a process of idealisation and abstraction. A mathematical collective obeys the *Axiom of Convergence*. This axiom states that the limiting relative frequency of any attribute does exist. Von Mises (1964, p.12) derives it from generalisation and abstraction of what he calls the *'Primary Phenomenon of the theory of probability'*: "experience has shown that in the game of dice, as in all the other mass phenomena [...] the relative frequencies of certain attributes become more and more stable as the number of observations is increased". And the *Axiom of Randomness*: stating that the limiting relative frequency of each attribute in a given collective is the same in any infinite sub-sequence of the collective. Each attribute of a given collective has a limiting relative frequency insensitive to 'place selection' where place selection is a rule for selecting a subsequence of a sequence in which the decision whether to retain the nth element does not depend on the value of that or any subsequent element of the sequence (Von Mises, 2014, pp.9-10). He considers that the axiom of randomness is also derived by a process of idealisation and abstraction of the observable fact of the failure of gambling systems e.g.the gambling system that claims that 'if the roulette stops on black then bet for three consecutive times on red and then five consecutive times on black again'. Allow calling this the '*Primary Phenomenon of Randomness*. The failure of gambling systems is, for von Mises (1964, p. 25), an empirical fact: "the authors of such [gambling systems] have all [...] had the experience of finding out that no system is able to improve their chances of winning in the long run, i.e., to affect the the relative frequencies with which different colours or numbers appear in a sequence selected from the total sequence of the game". Although von Mises does provide empirical support for his 'empirical laws', these empirical laws are only concerned with empirical collectives. Under what conditions one is legitimate to represent an *empirical collective* in terms of a *mathematical collective* is a subtle matter.

Von Mises is clear that his opinion on the matter aligns with the operationalist/conventionalist tradition in respect to mathematical science:"the relative frequency of the repetition is a 'measure' of probability, just as the length of a column of mercury is the 'measure' of temperature (von Mises, 1964, p.vi). And, that: "infinite collectives can be applied to finite sequences of observations in a way which is not logically definable, but it nevertheless sufficiently exact for scientific practice. The relation of theory of observation is in this case essentially the same as in all other physical science (von Mises, 1964, p.85). One way to interpret these passages is that, unlike Kolmogorov, von Mises' aim was not a purely mathematical theory of probability but rather, a theory of probability as a *mathematical science* understood along the lines of operationalism/conventionalism in philosophy of science. Von Mises (1964) limiting frequency theory is meant to be a purely empirical approach to probability where probability is operationally reduced to a measurable quantity i.e. relative frequency.

Nevertheless, at the core of operationalism/conventionalism lies the idea that for a theory to become a physical theory and not just pure mathematics, an operational definition of its basic terms in terms of *observables* is required. Gillies (2012) notes that von Mises' definition of 'probability' is not an operational definition in terms of observable relative frequencies since the definition includes limits in infinite sequences. While this is indeed the case, von Mises thinks that this is the only way to make a mathematically accurate theory of probability. This is a reasonable assumption especially since his aim was to provide a scientific theory of probability and as (Galavotti, 1997) observes, idealised limiting concepts can be found to other physically respectable notions such as velocity or density.

Especially when considering that von Mises' aim is a probability theory as a *mathematical science* one can evaluate his theory in terms of the general operationalist/conventionalist approach to mathematical science. For example, we can consider that his aim is to provide principles of coordination that establish a correspondence between the *mathematical term* 'probability' and physical relative frequencies. The coordination can be thought to be established by his '*Primary Phenomenon of the theory of probability*' and '*Primary Phenomenon of Randomness*'. These are to be considered ‘presuppositions about physical reality’, ‘truths by convention’ or ‘definitions in disguise’ enabling the *coordination* between *mathematical probability* and the physical concept of *relative frequency*.

Yet, this reading does not provide an operational definition of ‘mathematical probability’ in terms of observable relative frequencies. Rather, it establishes ideal cases where actual relative frequencies can approximate in the *limit*. Also, it does not address what this study considers as the main problem of limiting frequentism; that is, the analysis is essentially modal.

2.2.1.5 Problems of Limiting Frequentsim

Some of the problems arising when considering limiting frequency as an analysis of general probability follow and where possible, responses are suggested (for a comprehensive discussion see Hájek (2009)):

Limiting frequentism faces a stronger version of the reference class, a problem that Hájek (2009) calls the ‘reference sequence problem’ . That is, not only probabilities must be relativised to a reference class but also to *a sequence* within the reference class. The reason is that the value of the limiting frequency of an event or attribute also depends on the ordering of the trials in the finite segment of the infinite ensemble. He illustrates his claim through the following example: consider an infinite sequence of the results of a coin toss whose initial segment happened to be H, T, H, H, H, H, T, H, T, T,... Suppose that the corresponding relative frequency sequence for heads that begins with 1/1, 1/2, 2/3, 3/4, 4/5, 5/6, 5/7, 5/8, 5/9,... converges to 1/2. Now, by suitably rearranging these results, one can make the sequence to converge to any value in the internal [0, 1] she wishes. The problem is that there is no clear answer why one should prefer, for example, the temporal ordering of the results over any other possible ordering. Thus, not only there are infinitely many reference classes the single-case can belong to but also there are infinitely many ways for ordering the events even if a reference class is arbitrary chosen. Each of the ordering can result to a different limiting frequency of the event under consideration. Maybe a limiting frequentist could follow von Mises (1964) and embrace this inability of the theory to meaningfully apply to single cases. If limiting frequenstism is not considered applicable to single-cases then the reference sequence problem does not even arise.

Also, according to infinite frequency interpretation, probability is always conditioned on the finite initial segment of the infinite ensemble used in its calculation. However, there is no logical relation between the limiting frequency of the initial segment and the limiting frequency in the infinite ensemble. It is always possible that future repetitions of the experiment will lead to sequences without point-wise convergent limiting frequencies. Maybe a way to circumvent this problem is to restrict the sequences limiting frequentism can be applied to; that is, by saying that it only applies to sequences whose limiting frequency does converge. Von Mises does so by requiring that his theory applies to sequences that form a ‘collective’; that is, to sequences that satisfy (1) his axioms of convergence, and (2) his axiom of randomness.

At worst, limiting frequency interpretation entails a departure from the empiricist doctrine which is supposed to be the backbone of frequentism. Recall that infinite frequentism considers probability relative to infinite references classes such that probability is defined as limiting relative frequencies of attributes or events in that infinite reference class. Thus an infinite number of trials are required in order to define such probabilities. Yet, in the case that the actual world does not provide an infinite sequence of trials of a given experiment, the limiting frequency interpretation becomes a hypothetical or a counterfactual frequency interpretation. That is, it says that the probability of an attribute X in a reference class Y is the value the limiting relative frequency of occurrences of X within Y *would* be if Y were infinite. The crucial problem is that the essential reference to the counterfactual notion *would* entail that a modal element has entered frequentism and consequently, entailing a departure from the empiricist doctrine. Building on this issue, Lewis (1994, p.477) raises the following objection to infinite frequentism as an analysis of probability: Recall that according to infinite frequentism, when one asks the probability of a certain event type that has finitely many instances in the actual world, they depart from the actual world and they consider counterfactual worlds, where the events of that type are instantiated infinitely many times. Yet, different possible worlds will have different limiting frequencies. Thus, to provide an answer to the question 'what the frequency would be if the event under consideration had infinitely many instances', one has to select from these possible worlds those that are closest to actuality. Hypothetical frequentism is not able to provide such an answer; it lacks the concept of 'closeness' between worlds.[18]

The modal element that enters limiting frequency interpretations–and the departure from the empiricist doctrine that this entails–collides with the main aim of this study. Namely, to provide a Humean (non-modal) interpretation of single-case probability.

2.2.1.6 Frequentism and the Single-Case Probability

Most objections against the long-run frequency interpretation are due to its inability to handle single-cases ((Popper, 1959), (Hájek, 1997), (Hájek, 2009)). This inability is called the 'problem of the single-case', which in turn leads to 'the reference-class problem'. This section suggests that

[18] Lewis' claim is that what makes some possible worlds closer to actuality must be something the possible worlds with the right limiting frequencies share with the actual world. But if that's the case, then it's *that something* in the actual world that provides the answer. Lewis' idea is that this *something* is provided by the probabilistic laws of nature in the actual world. This study does not examine whether Lewis' 'Best System Analysis' can provide an analysis of 'general probability' for two reasons. First, Lewis (1994, p.475) considers the 'Best System Analysis' as an analysis of chance and that "chance is objective single case probability". Based on this, Chapter 1 followed Ismael (2011) suggestion and considered frequency and long-run propensity interpretations as competing interpretations of the concept of objective general probability, and single-case propensity interpretations and Humean reductions as competing interpretations of the concept of single-case probability. Second, the Best System Analysis seems that it uses frequentism-type reasoning to be able to derive chances (singe-case probabilities) out of the patterns in the Mosaic to begin with. More precisely, the probabilistic laws are supposed to provide some sort of statistical description of actual outcomes. In this sense, the actual occurrences of actual events;that is, the actual frequencies, appear to play a fundamental conceptual role in the Best System Analysis of chance. If this is indeed the case, then the Best System Analysis of chance seems to require an actual frequency conception of general probability to begin with.

these objections are of no real threat to long-run frequentism *if* we consider that the concern of frequentism exclusively lies with the concept of general probability.

Recall that for frequency interpretations, probability is a property of a reference class and not of a property of a particular event of the reference class. For actual frequentists, 'probability of heads equals half' means that in a particular reference class of coin tosses, the outcome heads appears half of the times. Nothing can be said regarding the probability of a particular event of that reference class other than that 'heads' either occurs or it doesn't; that is, its probability is either 1 or 0.

Regarding this problem, von Mises (1964, p.15) is clear that his concept of probability is not applicable to single case (token) events: "probability cannot be applied to this problem any more than the physical concept of work can be applied to the calculation of the 'work' done by an actor in reciting his part in a play". He considers this a virtue of his interpretation and thus his maxim : "first the collective–then the probability" (von Mises, 1964, p.17). That is, without the collective (reference class), probabilities cannot be defined. On the very first page of his seminal work, von Mises (1964, p.1) states his philosophical foundations through Lichtenberg's aphorism that "all of our philosophy is a correction of the common usage of words". One way to interpret von Mises' philosophical view on 'single-case probability', is that he considers it to be meaningless; that is, for him, 'single-case probability' is an artefact of common usage of words that philosophy aims to correct.

Reichenbach (1971), the other prominent frequentist, suggests that probability of a single event is an elliptic mode of speech; viz. it has no meaning on its own but acquires a 'fictitious' one through the 'transfer' of meaning, so to speak, from the general probability–the frequency produced from a sequence of repeated occurrences–to a particular case of that sequence. Reichenbach is careful to note that the meaning of the single case will be *fictitious*. If it were not, then we would have a case of the fallacy of division; that is, inferring that since probability is a property of the sequence of events, then it is also a property of a particular instance of the sequence.

Also, the indispensability of a reference class from the frequency interpretations results to the 'reference class problem'. As mentioned, this problem only appears if one considers frequentism to be applicable to single cases. In brief, this problem appears when one intends to assign probability to a single event but that event can be categorised in different ways and its probability may change depending on which reference class one considers it belongs to. Each event can belong to an infinite number of reference classes, and there is no indication that there is a unique class that can be considered ideal. There is no principle for determining a uniquely correct answer regarding the 'correct' reference class to calculate the probability from.[19] Yet, for the formalist

[19] Early frequentists attempted to tackle this problem. Venn (1888) gives the example of John Smith, a consumptive Englishman at age 50 and asks the probability of him living until the age of 61. He suggests that the probability of 'John Smith living to age 61' is the frequency of all people *like John Smith* who are 50 years old, relative to the frequency of all people *like John Smith* who live up to the age of 61. The problem is that it is unclear how to identify who the people 'like John Smith' are in a unique way. There can be many reference classes that John Smith belongs to i.e. consumptive Englishmen at aged 50 who smoke, consumptive Englishmen at aged 50 who drink

approach on the task of 'interpretation of probability' this study adopts, the 'reference class problem' is to be expected.[20] Gyenis and Rédei (2014, p. 2) in a relevant discussion note that: "Our freedom to choose measure theoretic isomorphic probability theories to describe the same random phenomenon manifests the conventionality of naming random events in probabilistic modelling". For the formalist approach, the 'reference class problem' appears to be a consequence of the fact that probabilistic assertions are derived by *applying* the mathematical theory of probability to non-mathematical phenomena and when it comes to *applications* of probability theory, conventional choices–including that of the reference class–are involved.[21]

To conclude, the clash of actual frequentism with our intuitions about probability appears to be mostly due to its inability to handle single-cases. This need not to be a problem if we distinguish between the concepts of general probability and single-case probability and consider that the concern of long-run frequentism exclusively lies with the concept of general probability. In that case, one should not expect frequency interpretations to capture our intuitions about single-cases to begin with. In this sense, an understanding of the concept of general probability in terms of long-run frequentism no longer appears that problematic. I suggest that the concept of general probability is captured in a long-run frequentism version of von Mises (1964, p.17) maxim "first the collective-then the probability"; that is, first a reference class of *actual* events, then the general probability–aka long-run frequency.

2.2.2 Long-Run Propensity

Long-run propensity interpretations define probability as the property of an experimental set-up to produce certain frequencies when the set-up is often repeated. We focus on Gillies (2000) long-run propensity interpretation of 'probability' that attempts to de-operationalise von Mises' limiting frequentism (Okasha, 2002). The main difference of Gillies' interpretation with that of von Mises is that he does not associate limits to infinite sequences directly with the definition of 'probability'. Instead, he builds on Popper (1959) and associates probability with 'generating conditions'. He breaks down Popper's propensity interpretations into early Popper (1959) long-run propensity where propensity does not apply to single , and into late Popper (1990) single-case propensity where propensity applies only to single cases.[22] He notes that by associating propensity with repeatable conditions it becomes unclear how one can go from repeatable conditions to the single case, concluding that Popper's long-run propensity does not account for single-case probabilities any more than frequentism does. Gillies notes that his aim is, similar to that of

alcohol, consumptive Englishmen at aged 50 who smoke and drink alcohol, consumptive Englishmen at aged 50 whose favourite colour is green, and so on. Many of these reference classes may result to different relative frequencies and consequently to different probabilities for the event "John Smith living up to the age of 61'.

[20]See discussion in Chapter 1, Sec. 1.2.

[21]The conventionalist/operationalist approach in mathematical sciences highlights that a choice of certain parameters over others is involved when applying *formal theories* to *physical phenomena*.

[22]I adopt Gillies' terminology and his distinction between early Popper's long-run propensity and late Popper's single-case propensity throughout

von Mises, a scientific theory of probability and he considers that ‘single-case probability’ is inescapably a metaphysical issue.[23]

From early Popper’s interpretation, I only focus on the features that Gillies adopts for his own long-run propensity interpretation. Popper (1959) considers propensities as properties of the experimental arrangement characterising the ‘disposition’ of the experimental arrangement producing certain frequencies when the experiment is *often repeated*, as a ‘probabilistic causal tendency’ associated with a long series of repeatable conditions resulting to frequencies equal to probability:

> [P]robability applies to sequences characterised by a set of generating conditions, a set of conditions whose repeated realisation produces the elements of the sequence [...] [These conditions are] “endowed with a tendency or disposition or propensity to produce sequences whose frequencies are equal to to the probabilities (Popper, 1959, pp.34-35).

More precisely, he suggests the following modification to frequency interpretations:

> The frequency interpretation always takes probability as relative to a sequence which is assumed as given; and it works that a probability is a *property of some given sequence*. But with our modification, the sequence in its turn is defined by its set of *generating conditions*; and in such a way that probability may be *a property of the generating condition* (Popper, 1959, p.34) [Emphasis in the original].

He clarifies that:

> [W]e we do not assume that a possibility as such has any tendency to realise itself; but we do interpret probability measures, or ‘weights’ attributed to the possibility, as measuring its disposition, or tendency, or propensity to realise itself [...] (Popper, 1959, pp.36-37).

One way to interpret early Popper is that he proposes that one can go from von Mises’ concept of frequentism to that of ‘propensity’ in the following manner: in frequency interpretations the sequence that generates the collective itself is included in the definition of probability. On the other hand, the move to long-run propensity allows probability to be attached to the generating conditions as well as the corresponding sequences, the collectives, they create. Thus, for early Popper propensity is not considered a property of the sequence resulted from the repetition of an experiment as in the case of frequentism. Rather, it is a property of the experimental setup to produce certain outcomes with certain frequencies when the experimental setup is often repeated.

[23] As I discuss in Chapters 3 and 4 I fully agree with Gillies on this.

One of the main reasons that Gillies (2012) introduces his long-run propensity interpretation is to distinguish his theory from the operationalist tradition that accompanied the frequency interpretation. Instead, he considers, similar to early Popper, relative frequencies as evidence for propensities and as not part of the definition of 'probability. In developing his own long-run propensity interpretation, Gillies (2000, p.823) takes the passage of Popper (1959, p.55) that "we have to visualise the conditions as endowed with a tendency, or disposition, or propensity, to produce sequences whose frequencies are equal to the probabilities" as part of his own version of the propensity theory with the exception that he requires for frequencies to be only approximately equal to propensity.[24]

A way to interpret Gillies is that propensities depend on the setup of a certain experiment which when often repeated, produces certain frequencies. The frequency in which an outcome occurs is a measure of the 'propensity' of the experimental setup to produce such outcome. For instance, consider a die tossed for n number of times where the actual frequency of '3' in n trials equals 1/6. According to the long-run propensity interpretation, the relative frequency of '3' is a measure of the propensity of the experimental setup e.g. the die, the manner in which it is thrown etc., to yield outcome '3' when the particular experimental set up is repeated for n number of times.

In particular, Gillies considers one of the main problems of frequency interpretation to be that their definition of probability cannot be 'methodologically' falsified, that it lacks what he calls a 'Falsifying Rule of Probability Statements' he appreviates as FRPS. He distinguishes between 'strict falsification' and 'methodological falsification' stating that while probability statements cannot, even in principle, be strictly falsified–for the same reason that they cannot be verified–they can still be considered as practically falsifiable statements. Following Popper, he calls this claim the 'methodological falsifiability of probability statement'. This rule essentially states that a statistical statement or hypothesis should be considered to be falsified if the statistical test lies in the certain rejection region (Gillies, 2012, p. 147). This is how he distinguishes his long-run propensity interpretation from frequency interpretations:

> In the frequency theory the link between probability and frequency was established by giving an operationalist definition of probability in terms of frequency. In the present version of the propensity theory the link is established instead by adopting the falsifying rule.

From this he argues that:

> With the help of FRPS, we can derive from probability hypotheses results about frequencies, and these can be checked by observation (Gillies, 2012, pp.149-150).

[24] For the original remark of Popper see Popper (1959, p. 35).

Before discussing how Gillies theory results to Euthyphro-type problems consider first focusing on the role that FRPS plays in his long-run propensity just to suggest that it is not clear why he thinks that the operationalist nature of frequency theories of general probability does not allow for a rule that plays essentially the same methodological role with that of FRPS.

Consider the following response that a frequentist may give to Gillies' proposed long-run propensity theory: Probability statements cannot, even in principle, be strictly verified but they can be 'practically verified'; a probability statement or hypothesis is said to be practically verified if the statistical test *does not* lie in the certain rejection region. Call this the 'Verifying Rule of Probability Statements' (VRPS). If the FRPS is considered sufficient for practically falsifying probability statements, then it is hard to see why the VRPS would not also be sufficient for practically verifying probability statements. The former states that a statistical statement or hypothesis *should* be considered as falsified if the statistical test lies in the certain rejection region. The latter states that probability statements can be considered 'practically verified' when the statistical test *does not* lie in a certain rejection region. I reckon, if the FRPS justifies the statistical practise so does the VRPS. Then, the ultimate difference between frequency and long-run propensity theories of 'general probability' cuts down to whether one considers falsificationism or verificationism/operationalism as the proper method of scientific inquiry.

2.2.3 Long-Run Propensity and Euthyphro's Problem

A reason to favour frequentism over long-run propensity interpretations of 'general probability' is that the latter face *Euthyphro's Problem*; that is, when Plato has Socrates asking Euthyphro whether: *Is the pious loved by the gods because it is pious, or is it pious because it is loved by the gods?*. In the context of long-run propensities: Are the *frequencies* stabilised because of the *propensity*, or is it a *propensity* because it stabilises *frequencies*?'. If one supports that the stabilised frequencies produced by a long series of repeatable conditions do so *because* of propensity, its status as propensity cannot be due to the stabilised relative frequencies produced. In other words, if the definition of propensity makes essential reference to stabilised relative frequencies then the analysis becomes circular–the frequencies produced are the evidence of propensity and its status as propensity makes essential reference to the relative frequencies produced.

To illustrate this, let's consider what the long-run propensity interpretations remark about the probability of 'a fair coin lands heads'. It claims that it is due to the propensity of the coin-tossing set-up that when the set-up is often repeated it results to a sequence with stabilised frequency of the outcome 'heads' (approximately) equal to 1/2. Thus, the experimental set up 'fair coin toss', when repeated many times, has a propensity to realise outcome 'heads' (approximately) equal to 1/2. It is indeed an observable fact that actual repetitions of the experimental set up result to a stabilised frequency of 'heads' (approximately) equal to 1/2. Nevertheless, actual repetitions remain silent regarding the reasons why their outcomes are stabilised.

In general, the main problem with long-run propensity theories is that they introduce the distinct concept of 'propensity' to interpret 'general probability' with, but the concept of 'propensity' makes essential reference to relative frequencies. In consequence, long-run propensity interpretations appear to be frequency interpretations in 'disguise'.

2.3 'Single-Case Propensity'

This section discusses *Single-case propensity* interpretations of the concept of single-case probability. It distinguishes between two types of single-case propensity interpretations: *Hard propensity* and *Hybrid propensity* interpretations. Hard propensity interpretations are considered those exclusively concerned with 'single-case probability' such as the interpretations of late Popper (1990), of Miller (1995) and of Giere (1973b). On the other hand, Hybrid propensity interpretations of 'single-case probability' are considered those that make essential reference to the concept of 'general probability'; the focus is the interpretations by Mellor (2004) and Suárez (2013). It is argued that none of the single-case propensity interpretations discussed provides a coherent analysis of the concept of single-case probability.

2.3.1 Hard Propensity Interpretations

Generally, for *Hard Propensity* interpretations, propensity is considered as a property of an experimental arrangement, of a physical system, to realise a *particular* outcome on a *particular* occasion. For instance, the propensity of a *particular* experimental arrangement of 'a coin toss' to realise a particular outcome e.g. 'heads'. Importantly, propensity is not considered a measure of frequencies but rather a measure of the states of affairs constituting the *token* under examination to realise a *particular* outcome on a *particular* occasion. Thus, propensities are considered to apply to *tokens*.

Popper (1990, p.17) considers that: "propensities in physics are properties of the whole physical situation and sometimes of the particular way a situation changes". In this sense, propensities are to be understood either as causal factors or as the product of the operations of causal factors to produce a *particular* result on a *particular* occasion. Similarly, Miller (1995, p.138) considers that propensities are instantiated in the complete situation of the universe at a time: "Propensities depend on the situation today, not on other situations, however similar. Only in this way do we attain the specificity required to resolve the problem of the single case". Thus, both late Popper and Miller do not claim any relation between propensity and relative frequency. We can infer from this that they were aiming exclusively for an interpretation of the concept of single-case probability.

In a similar spirit, Giere (1973b) defines 'probability' as a propensity, as a 'tendency' of experimental set ups as a whole to produce various outcomes in *particular* trials. These 'tendencies' are considered as some type of modal connections that 'incline' towards various outcomes on particular trials (Giere, 1973a). Giere's interpretation of 'single-case probability' can be summarised along the following lines: propensity is the 'tendency' of an experimental set up as a whole hence whenever the set up is instantiated propensity is instantiated as well. For example, in a coin-tossing set up, propensity is the 'tendency' of the set up–the coin, the tosser, gravity, magnetic fields etc.–to generate outcome 'heads' with degree half. When the set up is instantiated the same does its propensity.

However, as Hájek et al. (2011) note this does not reveal however what propensity *is*. Giere (1973b) attempts to justify 'propensity' as a reputable scientific concept by drawing an analogy between propensity and electric charge suggesting that propensity is linked to the physical quantities of the experimental set up in the same manner that electric charge is. Since, he argues, 'electric charge' is considered as a reputable scientific quantity then the same is the case with propensity. Hitchcock (2001) notes that the analogy between propensity and electric charge is deceptive: one can learn about the properties of 'electric charge' by empirical means e.g. that it can be of two types, that positive electric charges repel whereas opposites attract etc. It is unclear how one could figure out whether propensities are non-negative, normalised and additive etc.

This is a problem that all hard propensity interpretations face: they all introduce a separate entity, the propensity, but they do not inform us what propensity *is*. Consider the following analogy: An experimental set up involves a gas and the equipment enclosing the gas. The pressure of the gas is indeed a property of this set up. However, the fact that the pressure of the gas is a property of the set up does not enlighten us regarding what gas pressure is. Rather, 'gas pressure' is defined–independently of the fact that it is a property of the set up–as a measure/description of the gas molecules microscopic collisions with the surface of the container of the gas. Similarly, an experimental set up is such that it has a numerical property called 'propensity'. Nevertheless, that propensity is a property of the set up, it does not inform one what propensity *is*.

On the one hand, hard propensity interpretations of 'single-case probability' cannot be of any real help for one whose aim is to make sense of assertions such as 'the probability of *this* coin tossed as *this* time, landing head is half'. Interpreting 'single-case probability' as a propensity would be of help if one knew what propensity is. On the other hand, this may not be considered as a serious problem for the advocates of these interpretations. They already 'entangled' propensity with two heavy metaphysical commitments. *First*, a conviction to 'metaphysical indeterminism' that states that the world is not fixed in advance, that: "the world has a propensity, which is neither cast-iron necessity nor cast-iron impossibility, to develop in the way described" (Miller, 1995, p. 139). *Second*, an ontological commitment to modalities: "the modal element in the propensity interpretation cannot be dismissed as idle metaphysics, for without it there is no objective interpretation of single-case probability at all" (Miller, 1995, p. 139).[25]

[25] Chapter 3 argues that the direct association of 'propensity' with 'indeterminism' creates more problems with the coherency of hard propensity interpretations.

2.3.2 Hybrid Propensity Interpretations

'Hybrid propensity interpretations' are considered the interpretations of 'single-case probability' that make essential reference to the concept of 'general probability' as frequency. This section discusses the Hybrid propensity interpretations of Mellor (2004) and Suárez (2013). It argues that, similarly with long-run propensity interpretations, they face Euthyphro's Problem as well.

Beginning with the interpretation of Mellor (2004), who considers propensity to be fixed by the physical properties of an *object*. He considers propensities: (1) to supervene on the other properties of a repeated experimental setup and (2) to be responsible for the distribution of the resulted outcomes. For example, the outcome 'heads' in a 'coin tossing' experiment has a propensity 1/2 if and only if the association between the coin as the bearer of propensity, along with the other factors of the experimental set up e.g. angle of the tossed coin, gravity etc., generates the occurrence of outcome 'heads' with degree 1/2.

Mellor's propensity interpretation heavily relies on his characterisation of dispositional properties as enduring entities "identified through apparent change as unchanging bearers of changing properties" (Mellor, 2004, p.63). To describe his characterisation of dispositional properties he uses the example of 'fragility'. 'Fragility' is the dispositional property of a glass whether or not it is being, it has been or it will ever be dropped. The 'breaking' is the characteristic property of the trial 'dropping a glass' and it is common to all trials of the same kind. That is, breaking is a property of all the droppings of fragile glasses. Thus, glass breakage is explained through its dispositional property of fragility. Mellor (2004, p.63) takes dispositional properties,propensities included, as counterfactually true. That is, if an entity [v] e.g. a vase, in an experimental setup [D] e.g. 'dropping the vase', has the dispositional property [F] e.g. 'fragility', at a given time t, then it is also true that if v were a part of setup D at t, v would have had property F. In general, a dispositional property F is being displayed in an event (or experimental set up) D while the event D that bears F is a trial.

Importantly, Mellor distinguishes between propensities and deterministic dispositional properties e.g. 'fragility', on the basis that *propensities* are displayed through *chance distributions* over a set of possible events because they are not invariable in their outcomes. That is, propensities are displayed through chance–essentially frequency–distributions. Consider a coin having the propensity of 'fairness'. Such a coin will be called a chance setup as it is the conveyor of propensity of 'fairness'. The coin's 'fairness' is displayed through a chance distribution over the set of all possible outcomes e.g 'heads' and 'tails'. Such a display comes up via a chance trial like flipping a standard coin. On the other hand, deterministic dispositions are displayed through events instead of chance distributions because they are invariable in their outcomes e.g. 'fragility' is displayed invariably in all of its trials through breaking.

The first problem with Mellor's view concerns his claim that propensities are scientifically respectable dispositions. He sets the following requirement for a disposition to count as scientifically sincere: A disposition is scientifically sincere if and only if it is described on grounds different than the ones it purports to explain. These grounds are provided by the nomic connections between the disposition under examination and other dispositions already established as scientifically accredited. His relies this condition on his principle of connectivity: two physical systems cannot only differ in respect to a single property. The principle of connectivity states that a scientifically sincere disposition supervenes on other properties of the system in such a way that two systems identical in respect to all their physical properties cannot have different propensities.

Nevertheless, it is not clear why propensity counts as a scientifically sincere disposition as it is not independently described on grounds different than the ones it purports to explain; that is, the chance distributions. The display of propensities through chance distributions is the very fact that it distinguishes propensity from deterministic dispositions. This is another case of Euthyphro's problem: Are the chance distributions the way they are because of propensity? Or is it a propensity because the chance distributions are the way they are? The definition of propensity makes essential reference to the observed chance (frequency) distributions that it purports to account for. It is unclear whether Mellor's characterisation of propensity satisfies his own condition for a scientifically respectable disposition.[26]

The second problem with Mellor's view pointed out by Fenton-Glynn (2011) concerns Mellor's distinction between deterministic dispositions and propensities; that is, the reasons why Mellor considers deterministic dispositions as invariable in their display. Mellor's example of fragility, in a sense, indicates the opposite. Consider the situation–likely experienced by most of us–where a glass falls on a hard surface without breaking. The situation is, in all relevant ways, similar with past situations where similar glasses fell and broke. In other words, the set up where the glass does not brake counts as an appropriate trial of fragility. Suppose further that someone immediately picks the glass from the ground, imitates the previous situation, drops the glass but this time the very same glass breaks. Mellor argues that at the moment the glass fell and did not break; it lacked the deterministic disposition of fragility. It is crucial for Mellor to maintain that deterministic dispositions are invariable in their display. It is precisely what distinguishes between deterministic dispositions and propensities. This response places one in a *dilemma* with the following horns:

Horn 1: One can reject that deterministic dispositions are invariable in their displays. Rather, they display through chance distributions as well. For instance, the glass dropping on a hard surface has disposition of fragility with value 0.99...99. This accounts for the example where the trial did not result in glass breakage. Nevertheless, one would not be able to maintain the distinction between deterministic dispositions and propensities. One seems to be forced to conclude that *all dispositions are propensities*.

[26] For instance, consider the stipulation that Sherlock Holmes is the only one having the ability to solve 200 murder cases in one week. Hypothesis: If 200 unrelated murders were committed and were solved in a week's period, Sherlock Holmes solved them. Police reports that 200 different murders were committed last night. In a week's period all of them have been solved. Do the latter facts confirm the existence of Sherlock Holmes? I don't think so. The relation between propensity and its display through chance distributions is dangerously similar with this example.

Horn 2: One can infer that propensities are not displayed through chance distributions either. Rather, propensities are reducible to deterministic dispositions. Thus, they are displayed through events as well. A coin for example, instead of having the propensity of fairness displayed through a balanced [heads and tails] chance distribution, it has two deterministic dispositions: 'headness' and 'tailness'. When the coin lands heads, one infers that at that time, the coin lacked the disposition tailness but it had the disposition of headness and *vice versa*. The distinction between deterministic propositions and propensities ceases away. One seems to be forced to conclude that *all dispositions are deterministic*. I do not see how one could avoid both horns of the dilemma. This however seems essential for maintaining Mellor's distinction between deterministic dispositions and propensities in terms of the invariability and variability in their displays respectively.

Suárez (2013, p.63) takes a different route and attempts a justification of propensities by an inference to the best explanation argument along the lines of Pierce's pragmatism that ontological postulates are acceptable as long as they exhibit explanatory power. He writes about his propensity interpretation:

> Propensities do not manifest themselves as frequencies in infinite virtual sequences but as probabilities in every single experimental trial. This demands a tripartite distinction between the dispositional property of the system (the propensity), its manifestation or effect in each single trial (the probability), and the consequences in the long-run of the experiment (the frequency) [...] The single-case view, by contrast, explains every single trial as the exercise of the underlying disposition, which displays itself in a probability (Suárez, 2013, p.76).

Therefore, according to Suárez (2013), first comes the propensity which is a dispositional property of a physical system. This dispositional property manifests itself in the single case and its manifestation is the probability–what this study calls 'single-case probability'. Thus, (single case) probability is to be seen as an instance of the manifestation, an exercise, of the propensity. The consequences of the manifestation of propensity in the single case–the (single case) probability–is the observable frequency. The resulted frequencies reveal to the observer the different manifestations of the propensity–the 'possibilities'–as well as the 'strength' of each. For instance, consider the physical system of the toss of a coin that has the propensity of 'fairness'. This propensity manifests itself in the single case–the (single case) probability–which is an exercise of the propensity 'fairness'. The consequences of its manifestations are the frequencies that reveal to the observer that 'fairness' has two manifestations: 'heads' and 'tails' and each has the 'strength' of half. Based on these considerations, Suárez (2013) derives by an inference to the best explanation argument the existence of propensity as a 'sui generis property' which manifests itself in the single coin toss experiment, the repetitions of which have as a consequence the observable frequencies and thus explain them. From this he concludes that probabilistic displays are expressed as (single case) probabilities in every single experimental trial. He suggests that the stipulation of propensity in conjunction with the distinction between propensity, (single case) probability and frequency provides *important explanatory gain* such that the stipulation of propensity is justified.

In a nutshell, in Suárez (2013) theory propensity is considered an ontological postulate, a hypothetical entity, a ‘sui-generis’ dispositional property of the system that explains and by inference to the best explanation grounds the manifestation of propensity which is the (single case) probability. The repetitions of (single case) probability and the propensity that grounds it have as a consequence, and thus explain, the *observable frequencies*. For instance Suárez (2013, p.76) explains the radioactive decay in the single case as follows: “On the single-case view developed here the actual decay observed is adequately explained by the propensity invoked (the element’s ‘half-life’) and its display in the appropriate probability of decay within a given period of time”.

Given that the argument is an inference to the best explanation type of argument, one ought to be precise about what *needs* explanation and what *does* the explanation; that is, what is the *explanandun* and what is the *explanans*. Suárez’s interpretation allows the following interpretation indicating that his view faces the problem of Euthyphro: The explanation of ‘why particles are found to decay in the rate they do’ is *because* they have the propensity of ‘half-lifeness’. The reason that one knows that they have this propensity is *because* they decay in the observed rate. Also, the reason one knows that the particles decay at the observed rate is *because* they have the propensity of half-lifeness. Once again, it is known that they have this propensity *because* they decay in the observed rate and so on. But then, propensity is the *explanans* and frequencies the *explanandun* only for the frequencies to become the *explanans* that ‘explains’ the propensity which now has the role of the *explanandum*.

Suárez’s distinction between propensity and (single case) probability does not avoid Euthyphro’s problem: Recall that *propensity* grounds *(single case) probability* and together explain the observable frequencies. Then, the explanandum is the *frequencies* whose explanans is the *(single case) probability*. Then *(single case) probability* becomes from explanans the explanandun, whose explanans is the *propensity*. The explanans (the *(single case) probability*) and the explanandum (the observed *frequencies*) are ‘explained’ by the same thing, the *propensity*. Both *propensity* and *(single case) probability* reveal themselves in their joint consequences, the observed *frequencies*. It is questionable, to say the least, whether this is a satisfactory justification of ‘propensity’.

2.3.3 Humphreys Paradox

This subsection discusses Humphreys paradox arising from considering propensity as a ‘causal factor’ or as ‘the product of the operations of causal factors’. It is thought to impose a threat to the admissibility of single-case propensity interpretations that associate ‘single-case probability’ with causality. Because the ‘Humean Propensity’ analysis of ‘single-case probability’ introduced in Chapter 5 allows for a connection between ‘propensity’ and causality, this section describes

Humphreys Paradox and two ways that one can respond to that.[27] Here is how Salmon (1979) describes Humphreys Paradox:

> [T]here is an important limitation upon identifying propensities with probabilities, for we do not seem to have propensities to match up with 'inverse' probabilities. Given suitable 'direct' probabilities we can, for example, use Bayes' theorem to compute the probability of a particular cause of death. Suppose we are given a set of probabilities from which we can deduce that the probability that a certain person died as a result of being shot through the head is 3/4. It would be strange, under these circumstances, to say that this corpse has a propensity (tendency?) of 3/4 to have had its skull perforated by a bullet. Propensity can, I think, be a useful causal concept in the context of a probabilistic theory of causation, but if it is used in that way, it sees to inherit the temporal asymmetry of the causal relation (Salmon, 1979, pp.213-214).

In a nutshell, the paradox remarks that if propensities are causal factors and if probabilities are propensities then conditional probabilities must be conditional propensities. Yet, conditional probabilities are reversible in a way that conditional propensities (causal factors) are not. This results to paradoxical conclusions. For example, consider that the following probabilities are assigned to the events 'it snows' and ' a car accident happens': P(snow) = 0.1 and P(accident) = 0.4 and P (snow | accident) = 0.9. By Bayes' rule one can reverse the conditional probability of P (snow | accident); that is, P (snow| accident) = P(accident| snow) * P(snow) / P(accident) = $0.9 * 0.1/0.4 = 0.255$. If probabilities are causal factors–propensities–then it seems that the 'car accident' has a substantial causal effect on the 'icing of the roads' since the P(snow| accident) > P(snow) as if the effect raises the probability of it's cause.

Along the lines of Humphreys paradox, Milne (1986) challenges the epistemic usefulness of conditional probabilities when probability is defined as a propensity. Consider his argument through an example: There is an urn with ten balls, five blue and five white. The blue balls have the following integers written on them: 10, 20, 30, 40, 50 while the white ones the following decimals: 0.1, 0.2, 0.3, 0.4, 0.5. Consider the question: 'what is the $P(30|blue)$?'. The intuitive answer is 1/5. But if probabilities are causal factors, $P(`30'|blue)$ can only be either 1 or 0. That is, if probabilities are causal factors describing causal links then the conditions that fix the probability 'blue' to be equal to 1 also render the outcome 30 incompatible with outcomes $10,20,40,50$. If each ball in the urn represents a unique causal branch, then the fact that the ball is blue eliminates five out of ten branches. We know as a fact that we are in one of the five mutually exclusive branches where the ball is blue. Being in one branch eliminates the possibility of being to any of others. Therefore, the probability that we are in the causal branch where the probability that the blue ball drawn from the urn has integer 30 ($P(30|blue)$) is either 0 or 1.

[27] Humphreys (1985) does not introduce this paradox as an objection to propensity interpretation but rather, to show that probability calculus cannot be employed in the description of propensities.

In other words, the ‘$P(blue) = 1$’ is interpreted as saying that one is in one of the five *mutually exclusive* causal branches.[28] In this sense, Milne (1986) suggests that propensity interpretation marks the outcomes of conditional probabilities as either certain or impossible.

On the other hand, Szabó (2001b) argues that Humphreys paradox does not impose a fatal threat to the admissibility of propensity interpretations for two reasons:

First, relations like ‘P(snow | accident) > P(snow)’ are necessary yet not sufficient conditions for causation; they do not ipso facto indicate causality. Consider the necessary and sufficient conditions for a causal relation by Reichenbach (1991): let A_t and B_t' be events ‘snows’ and ‘accident’ occurring at times t and t' respectively, and t' is later than t. Then, A_t is a cause of B_t' *if and only if* $P(B_t'| A_t) > P(B_t')$ and there is no event C that occurs at a time earlier than or at the same time with t, such as C screens off B_t' from A_t. C is said to screen off B_t' from A_t if $P(A_t|B_t'$ & $C) = P(A_t|C)$. There is a ‘spurious’ correlation when the relation $p(B_t'|A_t) > p(B_t')$ holds even though A_t is not a cause of B_t'. In this case, one concludes that the relation holds because both A_t and B_t' are caused by a third factor, C. In this sense, the relation ‘P(snow | accident) > P(snow)’ is considered a ‘spurious’ correlation explained in terms of the operation of a common cause C instantiated at some point in the common past of both ‘snow’ and ‘car accident’.

Second, the paradox arises from a general misreading of conditional probabilities that if $P(B) = 1$ then $p(A|B) = p(A)$. Fixing the set up such as $P(B) = 1$, renders $P(A|B)$ as a non well-defined concept; it does not correspond to a unique condition. For instance, in the urn example, the $P(30|blue)$ is not a well-defined concept since it does not correspond to a unique condition but to five. In other words, the conditional probability ‘$P(30|blue)$’ does not mean that the $P(30)$ is 1/5 when the ball that is drawn is blue; when $P(blue) = 1$ because in such scenario the conditional probability is not a well-defined concept, it does not correspond to a unique condition.[29]

Humphreys paradox does not seem to impose a great threat to an interpretation of ‘single-case probability’ that associates the concept with that of ‘causality’, at least not when the latter is understood in terms of Reichenbach’s Common Cause Principle.

2.3.3.1 Single-Case Propensities: Concluding Remarks

Wrapping up with the discussion of single-case propensity interpretations, this section has distinguished between two kinds of single-case propensity interpretations of ‘single-case probability’: Hard and Hybrid. When it comes to Hard propensity interpretations, it has been argued that their the main problem is that they introduce a separate entity/property–the propensity–to interpret the concept of single-case probability with, but fail to inform us regarding what propensity *is*. In

[28]The five branches are: (i) P(blue) = 1 because P(‘10’) = 1, hence P(‘30’) = 0, (ii) P(blue) = 1 because P(‘20’) = 1, hence P(‘30’) = 0, (iii) P(blue) = 1 because P(‘40’) = 1, hence P(‘30’) = 0, (vi) P(blue) = 1 because P(‘40’) = 1, hence P(‘30’) = 0 and (v) P(blue) = 1 because P(‘30’) = 1, hence P(‘30’) = 1.

[29]Szabó (2001b) argues that this conclusion is not only about conditional probabilities understood as propensities (causal factors) but for conditional probabilities in general. Recall that Kolmogorov defines the *conditional probability* of A given B by the ratio of their absolute probabilities: $P(A|B) = \frac{P(A \cap B)}{P(B)}$, given $P(B) > 0$, for every A and B in $\mathscr{F}$ and that this ratio is *undefined* if either or both of the unconditional probabilities are undefined or if $P(B) = 0$.

regards to Hybrid propensity interpretations, it has been argued that they are threatened with incoherency. One way or another, Euthyphro's problem comes into play and thus they face the accusation of circularity.

2.4 Conclusion

This chapter examined the 'standard' interpretations of the concepts of probability: *'subjective probability'*, *'general probability'* and *'single-case probability'*, or 'chance'. *Section 2.1* discussed the Classical, the Logical and the Subjective interpretations of 'probability'. It was argued that the Classical and the Logical interpretations run into serious problems and that the Subjective interpretation of probability is the most tenable. *Section 2.2* discussed interpretations of 'general probability' such as long-run frequency, infinite frequency and long-run propensity. It is suggested that actual frequency interpretation provides the most viable interpretation of the concept of general probability. *Section 2.3* discussed interpretations of 'single-case probability' distinguishing between Hard propensity and Hybrid propensity interpretations. It has been argued that none of them succeeds in providing a satisfactory interpretation of 'single-case probability'. Hard propensity interpretations do not reveal what propensity *is* whereas Hybrid propensity interpretations face Euthyphro's problem and thus the accusation of circularity.

Notwithstanding, I agree with Popper (1990), Miller (1995) and Giere (1973b) that metaphysical commitments are necessary for an analysis of the concept of 'single-case probability'. Where I disagree is that an interpretation of the concept of 'single-case probability' requires metaphysical convictions to irreducible modalities and/or indeterminism. To this end, the next chapter explores the connection between the interpretation of 'single-case probability' and one's metaphysical stand on modality *de re* and 'genuine' indeterminism.

Chapter 3

Indeterminism, Possibility & Single-case Probability

"You can't have it both ways. Either [a] concrete situation, no matter how described and no matter what was 'similar' to it [admits] two outcomes or only one."

— Belnap and Green (1994, p.369)

Typically, metaphysical theories of 'single-case probability' as irreducible propensity come with a commitment to indeterminism. As discussed in Chapter 2, propensity theorists like Popper (1990), Giere (1973b) and Miller (2015) maintain that if determinism holds in our world, then there are no non-trivial objective single-case probabilities in our world. They suggest that single-case probabilities are only features of an indeterministic world. Put another way, without indeterminism there are no single-case probabilities.

Yet, in the general metaphysical debate between determinism and indeterminism, Müller et al. (2018, p.2) observe that determinism has the dialectical advantage as it is positively defined, whereas indeterminism is perceived as a *negative* concept typically defined as the negation of determinism. As they comment, a positive characterisation of a metaphysical position leads to theory development, while a negative characterisation of position is merely defensive.[1] If we grant them this claim, a development of a propensity theory of single-case probability that presupposes indeterminism appears to first require a positive characterisation of the metaphysical position of indeterminism. More precisely, a metaphysical interpretation of the 'possibility space' over which the propensities are defined by. Otherwise, the concept of propensity will not be a well-defined concept.[2] To this end, Belnap and Green (1994) make a strong case that a positive characterisation

[1]Hoefer (2016) writes that metaphysical arguments on this debate "are not currently very popular [...] philosophical fashions change at least twice a century [...] [and] for the foreseeable future metaphysical arguments may be just as good [...] as any arguments from mathematics or physics". The focus of this chapter is on some of the metaphysical arguments for determinism (or not).

[2]Chapter 2 argues that this is indeed the case with many propensity theories of single-case probability.

of indeterminism requires ontological commitments to modalities. More recently, Müller et al. (2018) proposed a *temporal* rather than modal positive characterisation of indeterminism in terms of what they call 'real possibilities'.

This chapter raises some concerns regarding the possibility of a positive characterisation of the metaphysical position of indeterminism. More specifically, it argues that the characterisation of indeterminism by Müller et al. (2018) in terms of 'real possibilities' does not succeed since the concept of real possibilities makes essential reference to the *negation* of determinism. Also, considering Belnap and Green (1994) argument that a positive characterisation of indeterminism ultimately cuts down to the existence of irreducible modalities, it raises some challenges regarding the coherency of ontologies that include irreducible modalities. The main goal of the chapter is to suggest that, as long as the aim is a coherent propensity theory of single-case probability, there are good reasons to disentangle the concept from that of indeterminism. This paves the way for the Humean propensity theory proposed in Chapter 5 that disentangles propensity from indeterminism and, more generally, from irreducible modalities.

The chapter is structured into two main sections. *Section 3.1* discusses the concept of indeterminism by Belnap and Green (1994) and the features that are considered essential for its robustness. *Section 3.2* critically examines two ways of distinguishing between the 'actual' and the 'possible', a requirement for a coherent analysis of the concept of indeterminism. Firstly, it examines the distinction between 'actuality' and 'possibility' by looking at the Classical Possibilism reading of modality *de re*. Secondly, it examines that distinction in terms of the temporal concept of 'real possibilities' by Müller et al. (2018).

3.1 'Indeterminism'

Indeterminism is typically defined as the negation of determinism. A system or a world is deterministic only in the case where the state of the system or the world at one time fixes the state of the system or the world at all future times. A system is indeterministic if it is *not* deterministic. Earman et al. (1986, p.13) provide a more precise definition of determinism in terms of possible worlds semantics: Let W be the class of all physically possible worlds. The world $w \in W$ is deterministic if and only if for any world $w_i \in W$ it is the case that: if w and w_i are in the same state at some time t_0, then they are in the same state at all times t. The world w is indeterministic if it is *not* deterministic.

Belnap and Green (1994, p.365) in their penetrating article suggest a positive conception of indeterminism whose central idea is that "at a given moment in the history of the world there are a variety of ways in which affairs might carry on" and discuss the conditions of how the world must be like to have such a concept of indeterminism as its feature. They are specifically after a concept of indeterminism suitable for Branching theories which, as they note, must be *local*, *pre-probabilistic*, *objective*, *feature independent* and *de re* (Belnap and Green, 1994, p.367).[3]

'Indeterminism' must be *local* in the following sense: it must be ascribable to local transitions where a transition is any ordered pair of events where the first (the initial) event *entirely* precedes the second event (the outcome) in the causal order. For example, consider a situation where a coin has been tossed and it landed heads. The transition toss-to-heads is locally indeterministic if at the time of the toss the transition toss-to-tails is also possible. In contrast, if the coin was loaded such that the only possible transition was that of toss-to-heads then the transition was deterministic. In the latter scenario, it was not the case that at that moment in the history of the world there were a variety of ways in which affairs could carry on.

'Indeterminism' must also be '*pre-probabilistic*': consider a toss of a coin that landed heads and the claim that despite the fact that the coin has landed heads there was also the possibility that the coin could have landed i.e. tails. This assertion does not require numbers representing probabilities but only 'local possibilities'. As Belnap and Green (1994, p.368) nicely put it: "Any concept of probability must rest on a concept of possibility". As an illustration of this point, consider for instance the case of a coin toss that landed heads. One may say that when the coin was tossed there was 1/2 chance (or single-case probability) of it landing heads. But even before numbers are assigned to the situation there is the intuitive idea that despite the fact that the toss of the coin has landed heads, there was another possibility for the coin to land to a different outcome i.e. tails. At this early intuitive level there is no reference to something global e.g. a world, a theory, laws of nature etc., as part of the concept. Nor does the concept require numbers representing probabilities; the idea that there were two possibilities the coin could have resulted in does not require numbers representing the strength or the likelihood of each possibility. On the contrary, the concept of probability rests on the concept of possibility otherwise called 'possibility space'. In this sense, the concept of possibility seems to be more fundamental than that of probability. If that is the case, a metaphysical interpretation of possibility (or possibility space) over which the single-case probability is defined by needs to be provided first. Now depending on one's metaphysical reading of possibility space i.e. strict actualism, classical possibilism, modal realism etc., the concept of single-case probability defined in terms of that space changes substantially.

'Indeterminism' must be *objective*; that is, for a transition toss-to-heads to count as indeterministic the other possible transition toss-to-tails does not express anyone's beliefs. They write:

[3]They also require that the concept of 'indeterminism' is *existential* (at least some transitions satisfy the previous conditions) and *hard* (there is a rigorous theory for the concept of indeterminism). Branching theory satisfies the *hard* condition as it indeed provides a rigorous formal framework for representing the modal/causal structure of the world as a tree-like structure of histories, such that a common history can deviate to different branches. For a comprehensive discussion of Branching theories see Emerson and Halpern (1986) and Belnap (1992). If one holds that the *existential* condition is also satisfied because of quantum mechanics (sic.) then the satisfaction of the remaining conditions grounds the central idea of indeterminism that "at a given moment in the history of the world there are a variety of ways in which affairs might carry on" (Belnap and Green, 1994, p.365).

> We mean that the question of how many possible outcomes there were for a certain throw shall be classed with the question of how many ears on a certain Scottie, and contrasted with questions that are explicitly about who thinks what about what, and whether it is reasonable to do so [...] We are after a concept of indeterminism that does not put the number of possible outcomes of a certain throw in anyone's head, or make it relative to laws or theories, or have it depend on the status of a conversation, or depend on what people care about (Belnap and Green, 1994, p.368).

'Indeterminism' must be *feature independent*: The transition toss-to-heads is feature independent indeterministic in respect to the outcome 'heads' as long as the transition toss-to-tails was also possible before the transition toss-to-heads was actualised. That is, the particular 'Scottie had two ears'. In contrast, the same transition is feature dependent deterministic in respect to the outcome event 'the coin lands somewhere' because the initial event (the 'toss') always leads to the same outcome e.g. the coin lands somewhere.

'Indeterminism' must be fundamentally *de re*, it must be ascribed to the transition itself such that we:

> Disbar pretending to plausibility of contradictory phenomena via colourful redescriptions. You can't have it both ways. Either [a] concrete situation, no matter how described and no matter what was 'similar' to it [admits] two outcomes or [it admits] only one. Thinking de re prevents you from evading the problems of indeterminism (or determinism) by switching descriptions–as if you could change the number of ears on a particular Scottie by describing it as very like a whale (Belnap and Green, 1994, p.369).

Their requirement that 'indeterminism' must be *de re* indicates that they are after a modal–plausibly along the lines of classical possibilism–concept of indeterminism rather than a concept of indeterminism defined in terms of possible worlds as in modal realism or one defined in terms of the temporal concept of real possibilities that Müller et al. (2018) suggest.

The last two conditions are of particular importance; they say that 'indeterminism' must be ascribed to the transition itself irrespective of the description under which the initial event of the transition falls and irrespective of what we think about it. That is, irrespective of how we describe the 'tossing of the coin', if after the toss (the initial event) there are more than one possible outcome events the initial event of the transition could have evolved to, then the transition counts as indeterministic. What we are after is the ears of a certain Scottie. It is difficult to disagree with them. Either it is the case that at a given moment in the history of the world there are a variety of ways in which affairs may carry on *or*, that at any given moment of the history of the world there is only one way in which affairs may carry on.

Allow to consider that the conditions by Belnap and Green (1994) accurately specify how the world must be like to have indeterminism as its feature. A basic requirement of an ontology that has indeterminism as its feature is that it must be able to distinguish between the 'actual' and the 'possible'. That is, in order for 'indeterminism' to be ascribable to the 'toss of this coin', it must be the case that right after the toss there were–in the ontological sense and not in anyone's head and irrespective of the way the event is described–more than one possible local transitions e.g. toss-to-heads and toss-to-tails, out of which one has been actualised. As the concept of indeterminism requires a distinction between actuality and possibility, the following section focuses on two such theories that aim to do so.

3.2 Actuality & Possibility

This section describes two metaphysical theories that offer a distinction between the 'actual' and the 'possible'. First, the Classical Possibilism theory of modality *de re* and its irreducible modalities. Second, the theory of Müller et al. (2018) where the distinction between actuality and possibility is spelled out in terms of what they call 'real possibilities'. Before focusing on these two metaphysical theories, let me first briefly describe Strict Actualism or simply Actualism, the metaphysical theory on *modality de re* that rejects the ontological distinction between actuality and possibility and considers that alternative possibilities are always epistemic and arise due to lack of knowledge. According to Actualism, possibility is reducible to actuality. 'Possible' is something that either is or will become actual (Kneale, 1966). For this school of thought, only the actual things exist. If something is not actual then it is not. In a nutshell, the maxim of Actualism is that there is only one possible way states of affairs could carry on and this is the way states of affairs *actually* carry on.[4]

3.2.1 Classical Possibilism

Classical possibilism distinguishes between *being* (or *is*) and *existence* (or *actuality*). Accordingly, *being* (or *is*) is considered ontologically distinct from *existence* (or *actuality*). What *exists* is considered a small portion of what *is* such that everything that *exists is*, but not everything there *is exists* (Menzel, 2018). That is to say, there are things that *are* but do not *exist*; they are unactualised things or mere possibilities. *Existence* or (*actuality*) is considered as an intrinsic ontological property of *existing* things absent from mere possibilities. For instance, a person who had never been born, a 7 feet tall version of myself etc. lack the property of *existence*. Yet, as classical possibilism goes, 7 feet tall me could, if things had been different, have existed. By contingency, a 7 feet tall me does not *actually* exist but still *is*. This is of particular importance, mere possibilities could have existed in the physical world if they were actualised. Yet, even if they are never actualised these possibilities still *are*.

[4]There is vast literature on metaphysics concerned with modalities *de re*. We only focus on the views relevant for the concept of 'indeterminism' under examination. For a comprehensive discussion see Kneale (1966), Meixner (2006), Menzel (2018) and Yagisawa (2018).

Classical possibilism satisfies the conditions of 'indeterminism' in Belnap and Green (1994). It considers that all possible outcomes *are* in the ontological sense. Thus, besides the *actual* local transition from the initial event to the outcome event e.g. toss-to-heads, there were multiple *possible* transitions whose only difference from the actual one is that they lacked the property of *existence* (or *actuality*). Also, for Classical possibilism, mere possibilities *are* in the ontological sense and not in anyone's head. That is, all local transitions the initial event could have led to are ontologically on par with the one that has been actualised.

Quine's Challenge: Quine (1948) has famously argued that Classical possibilism is incoherent.[5] Consider the statement 'I have no dog'. However, it is possible that I could have had a dog. Therefore, there is a possibility that I have a dog and there is a possible being that is my dog. Classical possibilism remarks that my dog *is* but does not *exist*.[6] In effect, Quine's argument is that saying that there *are* things that *do not exist* is like saying that some existing things do not exist and this signals that an ontology that includes *mere possibilities* is incoherent. The 'standard' response that classical possibilists offer to the objection of incoherency is to substitute the distinction between *is* and *exist* with the distinction between *actual existence* and *possible existence*. Essentially, this response substitutes the ontological distinction between two modes of *being*: *is* and *exists*, with two modes of *existence*: *actual existence* and *possible existence*. The response claims that *actuality* is a property rather than an ontological mode that some existing things have while others lack, while maintaining that the metaphysical status of non-actual possible things is on par with that of the actual ones. Their only difference is that the former lack the property of *actual existence*.

Accordingly, Classical possibilism is reformulated as the thesis that there *exist* things that do not have the property of *actual existence*. As Yagisawa (2018) notes, it is unclear whether the revised version of classical possibilism actually responds to the core of Quine's objection or if it is just a re-labelling of the distinction between *being* and *is*. In effect, *being* gave its place to *possible existence* while *is* was replaced by *actual existence*. Everything else remains the same. Suppose, for the sake of argument, that classical possibilism is coherent (sic.). Even then, it is an inflationary ontology as it is hard to justify the exclusion from the realm of *being*, or from the realm of *existence* of anything that is not–what Kant calls–a contradiction in conception e.g. unmarried bachelors, squared circles etc. One way that classical possibilists have approached this problem is by considering 'thick' laws of nature that extend over and above actuality, restricting the space of mere possibilities.[7]

[5]In brief, Quine's criticism on possibilism is an elaborated version of Russell's theory of description. Russell (1905) with his theory of description aims to avoid ontological commitments to non-existing objects by eliminating references to objects for a quantification over classes of objects. Quine approached metaphysics in a similar way considering it to be about answering the question 'What is there?'; that is, what sort of entities exist. For Quine, ontological commitments are the things that the bound variable 'something' ranges over and as such there is no need for ontological commitment to universal entities.

[6]Quine argues that this raises unanswerable questions about the features of my dog like 'what breed is it?', 'Is it male or female?' etc.

[7]For such theories of the laws of nature see (Swoyer, 1982), (Shoemaker, 1998), (Bird, 2005).

In general, philosophers are divided when it comes to commitments to irreducible modalities a la Classical possibilism. For instance, Ladyman (2000, p. 854) considers that without commitment to modalities one ends up being a sceptic about science since she will “not believe in any objective facts about what would have happened had things been done differently”. On the other hand, Strawson (2008, p.25) considers such ontological commitments as “obscure and panicky metaphysics”. While my philosophical inclination is with Strawson, this is a metaphysical disagreement and each side can insist on their response. Notwithstanding, Quine’s challenge for the coherency of an ontology that includes irreducible modalities remains unanswered.

3.2.2 Real Possibilities

This section examines the novel attempt by Müller et al. (2018) to provide a positive characterisation of indeterminism in terms of what they call ‘real possibilities’. Instead of a modal distinction between ‘actuality’ and ‘possibility’, they suggest a temporal one where ‘actuality’ is substituted by ‘future epistemic possibilities’ and ‘possibility’ is replaced with ‘real possibilities’.

They propose that ‘indeterminism’ can be positively characterised as the existence of multiple ‘real possibilities’ for the future. The world is indeterministic if there are more than one ‘real possibilities’ for how the future can unfold, and it is deterministic when it lacks multiple ‘real possibilities’ for the future. They note that the feasibility of their account requires spelling out ‘real possibilities’ in a sensible way (Müller et al., 2018, p.4). This is what they write about ‘real possibilities’ and how these distinguish between the ‘actual’ and the ‘possible’:

> Real possibility is inextricably interwoven with the notion of time, and the relation between actuality and possibility is a temporal rather than a modal one [...] What is really possible in a given situation is what can temporally evolve from that situation against the background of what the world is like. At the core of the notion of real possibility, there is the idea that, unlike the present and the past, the future is not actual yet [...] Depending on whether the world is deterministic or indeterministic, in a concrete situation in time, there may be more than one possibility for the future, and each such possibility can be actualised. None of them is actual yet [...] What is really possible then varies from time to time: in the course of time, the range of real possibilities diminishes (Müller et al., 2018, p.3).

The account requires spelling out *in a sensible way* not only ‘real possibilities’ but also the distinction between ‘future epistemic possibilities’ and ‘real possibilities’. They consider the following example as a case of a “future epistemic possibility":

> When you buy a scratch card, you are most likely to win nothing. So if in fact you win nothing, you can refer to that statistical fact as an explanation. In this case, your use of statistics is merely epistemic: given the individual card, it is settled beforehand whether you will win or lose (Müller et al., 2018, p.2).

They consider quantum experiments as cases of 'real possibilities':

> There are however clear cases of *indeterministic explanations* that *do not* build on an underlying *deterministic base* [...] (W)hen a random bit is produced by letting a photon pass through a symmetrical beam splitter, the physical set-up allows for exactly two possible outcomes, with exactly 50:50 chance. The kind of indeterminism that is at stake here [...] [has] a clear mathematical formalism that describes the range of possible outcomes and their respective probabilities, and the whole theory is empirically thoroughly corroborated (Müller et al., 2018, p.2) [My emphasis].

Müller et al. (2018) suggest that their concept of 'real possibilities' is to be seen as a philosophical characterisation of the concept of indeterminism suitable for Branching theories. Belnap and Green (1994, p.1) consider that the concept of indeterminism suitable for Branching theories has as its central idea that at a given moment in the history of the world there are a variety of ways in which affairs may carry on. The concept of indeterminism in Branching theories together with their 'real possibilities', Müller et al. (2018, p.4) argue, allows for a *positive* definition of indeterminism "as the thesis that there is more than one real possibility for the future, whereas determinism is just the negation of indeterminism". More specifically, they define a situation as locally indeterministic if it corresponds to a branching point in the tree of histories. On the other hand, a situation is locally deterministic if there is only a single possibility for actuality to evolve. In other words, 'indeterminism' is defined from a local standpoint in time as the thesis that given the actual course of events up to now, there are alternative 'real possibilities' for the future.

They consider 'real possibilities' to be grounded in the nature of things such that: "what is really possible in a given situation is determined by what the objects can do in that particular situation in virtue of being the objects they are" (Müller et al., 2018, p.5). They require a notion of laws of nature that capture the idea that objects, through the manifestation of their 'potentialities', become causally efficacious and give direction to the possible future courses of events. Objects possessing these 'dispositional properties', 'essences', 'potentialities', 'powers' etc. are disposed to manifest them given the appropriate stimulus and they can do so in every possible world. Thus, one does not need to stipulate many possible worlds but only the real one. The real world is one with irreducible 'powers', 'essences', 'potentialities' etc. and if the world is indeterministic it has 'real possibilities' too. That is, their account rejects a categorical ontology for a dispositional ontology according to which the world is constituted of objects with 'irreducible dispositions'.[8] In this sense, the concept of real possibility is reducible to these irreducible 'powers', 'essences', 'potentialities' etc.

To summarise, one way to interpret Müller et al. (2018) account is as follows: (1) the concept of indeterminism in Branching theories, (2) their concept of 'real possibilities', and (3) a dispositional

[8]For an elaborated discussion on dispositional accounts of laws of nature see Vetter (2015) and for a discussion regarding dispositional ontologies see Bird (2005).

ontology allow one to provide a positive characterisation of 'actuality' and 'possibility' and thus of 'indeterminism' in terms of the temporal, rather than modal concept of 'real possibilities' as the thesis that there is more than one *real possibility* for the future.

I proceed by noting two problems with the characterisation of 'indeterminism' in terms of 'real possibilities' and in particular with the distinction between 'future epistemic possibilities' and 'real possibilities'. *First*, that the statistics of the quantum experiment Müller et al. (2018) consider clear cases of statistical explanations that do not build upon a deterministic base *do* admit statistical explanation that builds upon deterministic bases. *Second*, that the distinction between 'future epistemic possibilities' and 'real possibilities' makes essential reference to the *negation* of determinism; that is, that the basis of some statistical explanations *is not deterministic* and thus does not provide a *positive* characterisation of 'indeterminism'.

Problem 1: Consider again the examples they use to illustrate the distinction between future epistemic possibilities and 'real possibilities' respectively. On the one hand, an 'epistemic possibility' is when one buys a scratch card and wins nothing and infers to the statistical facts about scratch cards as the explanation of why she won nothing. The usage of statistics is epistemic because it has already been settled in advance–before buying the scratch card–whether she will win or not. On the other hand, a 'real possibility' is for example when a random bit is produced by letting a photon pass through a symmetrical beam splitter, where the physical setup allows for exactly two possible outcomes with the 'chance' of each possible outcome being exactly half. They consider this as a *clear* case of indeterministic explanations that *do not* build on an underlying *deterministic base*.

One way to interpret the claim that "when a random bit is produced by letting a photon pass through a symmetrical beam splitter, the physical set-up allows for exactly two possible outcomes, with exactly 50:50 chance" is that 'chance' is considered a by-product of an indeterministic world in a similar manner that 'standard' propensity theories interpret 'single-case probability' (Müller et al., 2018, p.2).[9] Yet, a direct association of 'single-case probability' with indeterminism in quantum mechanics makes the interpretation of 'single-case probability' circular–the interpretation of quantum mechanics depends on how one interprets probabilities (Home and Whitaker, 1992). Interpreting 'probability' in a certain way that fits our interpretation of quantum mechanics and then using such an interpretation of probability to justify our interpretation of quantum mechanics could only lead to circularities. As Home and Whitaker (1992, p.228) precisely put this: "it would be a fatal error to choose an approach to probability which suits our views on quantum theory, and then to use the former to justify the latter!".

All in all, it is not clear whether the probabilistic statistics one derives by performing quantum experiments (i.e. when a random bit is produced by letting a photon pass through a symmetrically

[9]Among the first to connect 'single-case probability' with a peculiar kind of probability, indeterminism in quantum mechanics, was Popper. Since then, following Popper's footsteps many interpretations of 'single-case probability', directly associate it with indeterminism in quantum mechanics (Popper, 1990), (Miller, 2015), (Ballentine, 2016), (Suárez, 2004), (Suárez, 2007), (Suárez, 2013).

beam splitter) is evidence or an explanation of indeterminism as Müller et al. (2018) state. To see why this is the case, consider Wallace (2019) argument that 'orthodox' (or 'standard') quantum mechanics consists only of the structural/mathematical core of the theory and the Born rule or probability postulate, whereas the 'Eigenstate-Eigenvalue link' (henceforth E-E link) and the 'projection postulate' (or the 'collapse rule') are parts of a proposed interpretation of quantum mechanics. The structural/mathematical core of orthodox quantum mechanics consists of three parts:

(1) States: A quantum system's possible states are represented by normalised vectors in some complex Hilbert space.

(2) Observables: To any physical quantity used to describe the system is associated a self-adjoint operator on that same Hilbert space.

(3) The state of a quantum system evolves over time according to Schrödinger's equation asserting that: $\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = -\frac{i}{\hbar}\widehat{H}|\psi(t)\rangle$ where $\widehat{H}$ is a self-adjoined operator corresponding to the system's energy.

The Born rule states: consider a given quantity O with an associated operator $\widehat{O}$ expressed as $\widehat{O} = \sum_i o_i \widehat{\Pi}(i)$ where o_i are the distinct values of the operator and $\widehat{\Pi}(i)$ projects into the subspace of states with eigenvalue o_i. In case that the quantity O is measured on a quantum system with state $|\psi\rangle$, then the only possible outcomes of that measurement are the eingenvalues o_i of the operator and the probability that the measurement results to o_i is $\mathrm{P}(O = o_i) = \langle\psi|\widehat{P}(i)|\psi\rangle$. These are the elements of 'standard' or 'orthodox' quantum mechanics.

The structural/mathematical core of the theory and the Born rule, Wallace (2019) argues, constitute the 'standard' or 'orthodox' quantum mechanics. If the E-E link is assumed to hold true, it results in ontic indeterminacy of certain quantum properties of certain states. The reasons are roughly the following: the E-E link states that a system possesses a determinate value of some quantity if and only if it is in an eigenstate of the operator associated with measurements of that property. More formally, the E-E link asserts that given quantity O with an associated operator $\widehat{O}$, a quantum system in state $|\psi\rangle$ has a definite value of O if and only if $|\psi\rangle$ is an eigenstate of $\widehat{O}$; that is, $\widehat{O}|\psi\rangle = o_i|\psi\rangle$. When this is the case, the definite value of the system in state $|\psi\rangle$ is the corresponding eigenvalue o_i. Yet, the structural/mathematical core of quantum mechanics entails that a quantum system in state $|\psi\rangle$ cannot be in a simultaneous eigenstate of position and momentum. For example, the complete precision of the position of a particle renders its momentum indeterminate and vice versa. To be more precise, given the E-E link the claim is not just that the exact values of both its position and momentum are unpredictable but rather, they do no exist. For instance, Salmon (1998, p.262) considers that the acceptance of the E-E link says that "the future behaviour of a particle such as an electron is not just unpredictable but [ontically] indeterminate".[10]

[10]In fact, there is an ongoing debate in literature whether the concept of ontic indeterminacy is coherent. For a discussion on the issue of the coherency of 'ontic indeterminacy' see (Evans), (Lowe, 1994), (Lowe, 1999), (Noonan, 1995) , (Hawley, 1998). There is no need to delve into this debate because even if quantum indeterminacy is ontic,

Nevertheless, this ontic indeterminacy of physical quantities is not a direct consequence of the standard quantum mechanics. Ontic indeterminacy holds if one assumes that the *interpretative postulate* of the E-E link holds true and even then, this entails ontic indeterminacy of certain properties of certain states of the quantum system.[11] Thus, ontic indeterminacy is not a consequence of the standard quantum mechanics understood as solely consisting of the structural/mathematical core of the theory and the Born rule.

Also, even if the E-E link is assumed to hold, the ontic indeterminacy that this implies does not suffice for concluding that quantum mechanical phenomena are indeterministic. In order to take the ontic indeterminacy of certain quantum properties as evidence for indeterminism, besides the E-E link, one needs to accept that the additional *interpretative postulate*, the projection postulate (the collapse rule) also holds true. The projection postulate asserts that given some quantity O with associated operator $\widehat{O}$ is measured on a quantum system in state $|\psi\rangle$ then the *act of measurement* gives rise to an *indeterministic* transition on the state such that right after *the measurement takes place* the quantum system is in one of the states: $\left|\psi_i\right\rangle = \frac{\widehat{\Pi}(i)|\psi\rangle}{\|\widehat{\Pi}(i)|\psi\rangle\|}$ and the probability that the quantum system transitions into state $\left|\psi_i\right\rangle$ is: $\mathrm{P}\left(|\psi\rangle \to \left|\psi_i\right\rangle\right) = \langle\psi|\widehat{P}(i)|\psi\rangle$.[12] Thus, to infer that quantum phenomena are indeterministic one needs to accept both interpretative postulates; both the E-E link and the projection postulate.

Yet, similarly with the E-E link, the projection postulate is not an indispensable part of standard quantum mechanics but merely an interpretative postulate.[13] If we remain within standard quantum mechanics' perimeters; that is, the structural part of the theory, and the Born rule, then it is not at all clear that the statistics of quantum experiments are clear cases of indeterministic explanations that do not build on an underlying deterministic base as Müller et al. (2018) claim. The fact that the Born rule enables the calculation of the frequencies of various outcomes when a particular type of measurement is performed does not reveal anything by itself about the physical process involved. Also, besides the indeterministic explanations of the statistics of quantum experiments i.e. the Copenhagen interpretation, other interpretations like that of Bohm and Everett provide *deterministic* explanations of the very same statistics.

This is enough to reject the claim of Müller et al. (2018) that the statistics derived by performing quantum experiments are *clear* cases of indeterministic explanations that do not build on an underlying deterministic base. The jury is still out.

Problem 2: The concept of 'real possibility' by Müller et al. (2018) intended to provide a positive characterisation of indeterminism for Branching theories makes essential reference to the negation of determinism. According to their account, what actually makes a real possibility 'really' real rather than a future epistemic possibility must be time plus *something else*. That is, a possibility is 'real' as long as it is not yet actualised plus *something else*.

ontic indeterminacy does not entail indeterminism (Hawley, 1998).

[11]Interpretations of quantum mechanics like the Bohmian interpretation reject the E-E link postulate.

[12]The projection postulate entails that quantum systems evolve under Schrödinger's equation only when a measurement is not taking place.

[13]The Everettian and the Bohmiam interetation of quantum mechanics reject this postulate.

This 'something else' seems to be the commitment that there are "clear cases of indeterministic explanations that do not build on an underlying deterministic base" (Müller et al., 2018, p.2). More specifically, their indeterminism thesis states that "there is more than one real possibility for the future" (Müller et al., 2018, p.4). A future possibility is thus considered 'real' rather than 'future epistemic' when its explanations "do not build on an underlying deterministic base" (Müller et al., 2018, p.2).

Thus, one has a 'real possibility' rather than a 'future epistemic possibility' when the basis of its statistical explanation *is not deterministic*. Since Müller et al. (2018, p.4) define indeterminism as the thesis that there is "more than one *real possibility* for the future", and consider a future possibility a 'real possibility' when the basis of the statistical explanations is *not deterministic*, then their characterisation of 'indeterminism' makes essential reference to the negation of determinism. That is, 'indeterminism' is when the basis of the statistical explanations is *not deterministic* .

The same problem arises if the distinction between future epistemic and real possibilities is spelled out in terms of the laws of nature: their indeterminism thesis states that "there is more than one real possibility for the future" (Müller et al., 2018, p.4). Considering future possibilities as real rather than epistemic if the underlying laws governing the distribution of statistics are *indeterministic* also begs the question.

3.3 Conclusion

This chapter has discussed the concept of local indeterminism by Belnap and Green (1994, p.1), the central idea of which is that "at a given moment in the history of the world there are a variety of ways in which affairs might carry on". It has considered two ways for spelling out the distinction between the actual and the possible: (1) Classical possibilism and its distinction between *is* and *being* or for its revised version between *actual existence* and *possible existence* and (2) the temporal rather than modal concept of 'real possibilities' by Müller et al. (2018). It has been argued that when actuality is distinguished from possibility in terms of ontic modalities a la classical possibilism, one faces Quine's challenge that an ontology that includes ontic modalities is incoherent, with his challenge remaining unanswered. Also, when the distinction between 'actuality' and 'possibility' is spelled out in terms of the temporal concept of 'real possibilities' as suggested by Müller et al. (2018), then the distinction makes essential reference to the negation of determinism and as such it does not suffice for a positive characterisation of the concept of indeterminism suitable for Branching theories.

The hope is that what has been said is sufficient to motivate the suggestion to disentangle the concept of single-case probability from that of indeterminism and, to that extent, to pave the way for the Humean Propensity theory proposed in Chapter 5 which disentangles 'single-case probability' from 'indeterminism' and more generally, from irreducible modalities. The next chapter discusses Humean ontologies, which avoid the problems of the distinction between the ‘actual’ and the ‘possible’ by reducing ‘possibility’ to an overarching actuality adopted for the remainder of this study.

CHAPTER 4

HUMEAN REDUCTION OF MODALITY *De Re*

"The point of defending Humean Supervenience is [...] to resist philosophical arguments that there are more things in heaven and earth that physics has dreamt of."

— Lewis (1994, p.474)

The previous chapter concluded with Belnap and Green (1994) suggesting that any concept of 'single-case probability' must rely on that of 'possibility'. Given that one of the main aims of this study is to provide an interpretation of 'single-case probability', this chapter delves into Humean reductions of modality *de re* where 'possibility', one way or another, is reduced to an overarching actuality. More specifically, for the 'Humean propensity' analysis of chance (Chapter 5) and the Humean *objective* reading of 'quantum probabilities' (Chapter 6). This chapter is divided two sections. *Section 4.1* briefly describes the metaphysical doctrine of 'Humean Supervenience' and the 'Humean Project'. *Section 4.2* draws a distinction between two types of Humean reductions of 'possibility', and suggests that this distinction leads to two different conceptions of the Humean project: first, 'Down to Earth Humeanism' standing for the metaphysical doctrine of Humean Supervenience and Actualism regarding modality *de re* and second, 'Possible Worlds Humeanism' which stands for the metaphysical doctrine of Humean Supervenience and 'Modal Realism' reading of modality *de re* by Lewis (1986b).

4.1 Humean Supervenience & The Humean Project

The Humean project aims to express *everything* found in the world in a manner compatible with the core of Humean metaphysics, stating that *truth supervenes on being*. Lewis has famously formulated the Humean project around the metaphysical doctrine of Humean Supervenience. In Lewis' words, Humean Supervenience expresses the idea that: "all there is to the world is a vast mosaic of local matters of particular fact, just one little thing and then another" (Lewis, 1987, p.ix). The doctrine of Humean Supervenience is divided into two distinct theses (Weatherson, 2016). Thesis 1 is considered necessary while Thesis 2 contingent:

Thesis 1: All the truths about the world supervene on the distribution of perfectly natural properties and relations in that world. Everything supervenes on the physical realm and can be expressed in physical terms. The concept of supervenience can be understood as the relation between two classes of properties. Let M and P be such classes of properties: M supervenes on P when there is no difference in M-properties without some difference in P-properties. M-properties cannot be altered unless P-properties are altered (Lewis, 1986b). This first thesis entails a metaphysical commitment to physicalism; everything that exists is physical.

Thesis 2: The perfectly natural properties and relations in the world are intrinsic properties of point-sized objects, and spatio-temporal relations. At the foundations of physical reality what we have is a mosaic of local particular matters of fact and regularities of occurrences without any necessary connection between them (Lewis, 1987, p.ix). This thesis is to be read as a feature of the 'mind-independent' world of physical reality. It says two things. *First*, that everything there is in the world globally supervenes on the microphysical domain.[1] This entails an ontological commitment to reductive physicalism.[2] In terms of the doctrine of Humean Supervenience, the microphysical domain is the distribution of intrinsic properties of point-sized objects and spatiotemporal relations. Fixing the point-sized objects of the world 'automatically' fixes everything, all the properties of that world. *Second*, it says that the fundamental space-time structure of the world is relativistic; that is, it is comprised of the four dimensions of space-time (Oppy, 2000).

The Humean project aims to express *everything* we find in the world in a manner compatible with the metaphysics of Humean Supervenience. This requires one to provide *truth conditions* for all *contingent* truths in terms of the Humean mosaic (Ismael, 2015). In other words, the Humean project aims to express every concept found in the world e.g. 'possibility', 'single-case probability', 'general probability', 'causality' and so on in terms of the Humean mosaic.

4.2 'Down to Earth' and 'Possible Worlds' Humeanism

I suggest distinguishing between two sorts of Humean Metaphysics in terms of their stand on modality *de re*: (1) 'Down-to-Earth Humeanism' and (2) 'Possible Worlds Humeanism'.

'Down to Earth Humeanism' stands for the doctrine of Humean Supervenience and of a strict Actualism regarding modality *de re*. The commitment to Actualism entails that the only form of

[1] One way to define global supervenience is the following: macro-properties globally supervene on micro-properties if and only if for any worlds w_1 and w_2, if w_1 and w_2 have an identical worldwide pattern of distribution of micro-properties, then they also have an identical worldwide pattern of distribution of macro-properties (McLaughlin and Bennett, 2018). For a comprehensive discussion of the concept of supervenience see Kim (2017).

[2] See Esfeld (2014) for a comprehensive discussion.

existence is the actual existence. Thus, it holds that alternative possibilities are epistemic and arise due to lack of knowledge. In short, 'Down to Earth Humeanism' and its project aims to express every concept found in the *actual* world including 'possibility', 'general probability', 'single-case probability', 'causality' and so on in terms of the Humean mosaic in a manner compatible with Actualism.

'Possible Worlds Humeanism' stands for the doctrine of Humean Supervenience and the 'Modal Realism' reading of modality *de re* by Lewis (1986b). According to Modal Realism, actuality is not an intrinsic property but a relational one: x is actual relative to y in so far as x and y are spatiotemporally related to each other, as long as they occupy the same possible world. Possible things are defined in terms of possible worlds; everything that exists in a possible world is reducible to its branches (or paths).[3] More specifically, Lewis (1970) considers actuality as a relational rather than an intrinsic property spelled out in terms of 'possible worlds'. All possible worlds are as real as the world 'out there'. *Our* possible world, *our* realm of actuality, is distinguished from the rest of possible realms (or possible worlds) in the sense that it is the maximal spatiotemporally connected whole we are parts of.[4] Something possibly but not actually exists when it is a part of a realm outside our realm of actuality; when it is a part of some maximal spatiotemporally connected whole, of a possible world we are not parts of. To say that there are things that are not actual is simply to say that there are things that do not occupy the same possible world with us. Everything that is for us actual is reducible to the possible world we are parts of.

It is important noting that the idea that possible worlds are spatiotemporally isolated from each other is in the core of Modal realism. The unification of every single possible world stems from the spatiotemporal interrelations of its parts such that "whenever two possible individuals are spatiotemporally related, they are worldmates" (Lewis, 1986b, p.70). This has implications when it comes to the concept of 'indeterminism'. Modal realism only provides a positive characterisation of 'global indeterminism': an event e.g. the toss of a coin, is said to be globally indeterministic when there is a possible world that shares causal history with the actual world up until the toss of the coin where the coin lands heads in the actual world and lands tails in the 'closest' possible one.

On the other hand, Modal Realism *does not* intend to ground 'local indeterminism'. Consider tossing a coin right now. Modal Realism does not aim to ground the claim that there exist a variety of ways the particular coin, tossed in the particular possible world we find ourselves to toss coins etc. may carry on. Within our realm of actuality, within the possible world we find ourselves tossing coins and performing quantum experiments there can be no possibilities. Alternative

[3]By considering actuality as a relational property Lewis avoids Quine's objection that an ontology that includes possibilities is incoherent.

[4]Later Lewis talks of 'analogical' spatiotemporally connected wholes. Lewis (1986b, p.75) writes: 'each world is interrelated by a system of relations which, if they are not the spatiotemporal relations rightly so called, are at any rate analogous to them'. For the purpose at hand there is no need to delve any deeper into this. For a detailed discussion see (Divers, 2006, pp.99-105).

possibilities require irreducible modalities. In summary, 'Possible Worlds Humeanism' and its project aim to express every concept found in the world including 'possibility', 'general probability', 'single-case probability', 'causality' and so on in terms of the Humean mosaic and in a manner compatible with Modal Realism.[5]

4.3 Conclusion

Taking into account Belnap and Green (1994) claim that any concept of 'single-case probability' must rely on that of 'possibility', this chapter has discussed two Humean reductions of 'possibility': *Down to Earth Humeanism* and *Possible Worlds Humeanism* which pave the way for the remainder of this study. To this end, consider a very brief summary of what has been discussed thus far:

Chapter 1 has considered that 'general probability' and 'single-case probability' are two distinct concepts, where 'single-case probability' is not defined in terms of 'general probability' nor vice versa. It has supposed that probabilistic assertions in physical theories e.g. statistical mechanics and quantum mechanics, require an objective interpretation–either 'general probability' or 'chance'–given that an agreement is reached on which objective concept is suitable for each case. Chapter 2 has argued that long-run frequency interpretation provides a satisfactory interpretation of 'general probability'. With the metaphysical doctrine of Humean Supervenience in our disposal, we consider the concept of general probability to supervene on token events of the Humean mosaic instantiating certain event types. On the contrary, it has also been argued that that none of the single-case propensity interpretations–hard and hybrid– provide a coherent analysis of 'chance'. Moreover, Chapter 3 argued that an interpretation of 'single-case probability' that directly associates the concept with that of indeterminism, especially in quantum mechanics, unavoidably ends up being circular: to the extent that the interpretation of quantum mechanics depends on one's interpretation of probability, if the interpretation of probability is also based on one's interpretation of quantum mechanics this could only lead to circularities.

The purpose of the remaining of this study is twofold. Chapter 5 seeks to propose an analysis of the concept of 'single-case probability' in terms of Humean propensities compatible with 'Down to Earth Humeanism' and 'Possible Worlds Humeanism' *and* with determinism (or not). Then, Chapter 6 aims to suggest that especially 'Down to Earth' Humeans have good philosophical reasons to categorise 'quantum probabilities' as *objective* 'general probabilities' rather than 'single-case probabilities' or 'chances'.

[5] Van Inwagen (2008) is accurate to note that Lewis' Modal Realism version of possibilism shares very little, if anything at all, with Classical possibilism reading of modality *de re*.

Chapter 5

Humean Propensities

> *"Then what is probability? And how is it possible then that physics and other empirical sciences apply a formal (mathematical) theory of probability, without noticing a problem arising from this unanswered fundamental question?"*
>
> — Szabó (2007b, p.3)

Some of our currently best physical theories e.g. statistical mechanics and quantum mechanics make chance assertions. This chapter is concerned with the following question: *What could the chance assertions of our currently best probabilistic physical theories possibly mean, given one's commitment to their objectivity?* Answering this question is important for at least two reasons. First, it is unclear how one can take the teachings of our currently best scientific theories–which supposedly describe how the world is–seriously if they do not know what their chance assertions mean. Second, one has to consent to a subjectivist reading of their chance assertions which as Miller (2015, p.125) notes, it collides with the *desideratum* of an objectivist theory of scientific knowledge.

The Humean propensity interpretation of the concept of chance this chapter aims to develop is proposed as a modest answer to the question above. This interpretation is a complementary reading of Lewis' Best System Analysis (henceforth BSA) of 'chance' and Szabó's 'no chance' interpretation. I call this interpretation 'Humean Propensity' because, as I claim, it captures the features of the hard propensity interpretations avoiding some of their problems while it does not deviate from the metaphysical doctrine of Humean Supervenience. The proposed interpretation makes four key claims. First, that chance is *partially* defined as a version of the BSA; the BSA is considered to 'fix the reference' of chance in a world yet we do not require for the BSA to reveal the nature of the quantity it refers to. Second, that the basic guide to (BSA) chance assertions is that of Bridgman; in a nutshell, 'Bridgman's guide' says that operational definitions have a *constitutive*, not necessarily exhaustive, role to play in the meaning of a given concept. Third, that Bridgman's guide to (BSA) chance assertions reveals that their truth-makers are the ordinary, already known and well-defined physical quantities of Szabó (2007b). I call them 'Humean propensities' since as I claim, they share many features of modal propensities. Forth, the *basic* meaning of BSA chance assertions corresponds to the *set of operations* that the Humean propensity in question is measured by. In brief, the proposed analysis suggests that the concept of BSA chance corresponds to ordinary, already-known, and well-defined physical quantities. The basic meaning of BSA chance is defined operationally; it corresponds to the set of operations the value of the chance assertions in question is measured by. In a nutshell, the Humean propensity proposal attempts to provide an operationalist analysis of the concept of BSA chance.

To be clear from the outset, the Humean propensity proposal has a catch. It does not claim to establish any fixed principle specifying the relation between the concepts of single-case probability (or chance) and that of rational degrees of belief (or rational credence); that is, it does not claim to establish that the concept of chance is or should be a 'guide to rational life'. I will suggest that as long as the aim is a non-circular analysis of chance, there are good reasons to accept that setback and still capture many of the intuitions we normally assign to the concept of chance, especially in the sciences.

This chapter is structured into four sections. *Section, 5.1* discusses the Lewisian account of chance, raising some concerns with its tenability. *Section 5.2* discusses two alternative proposals aiming to complement Lewis' account while avoiding some of its issues. These are Schwarz (2018) proposal, which I call the 'No-interpretation guide', and 'Bridgman's guide' that I instead propose. *Section 5.3* focuses on Szabó's physicalist 'no-probability' interpretation of probability ((Szabó, 2007b), (Szabó, 2010)). I call Szabó's interpretation 'physicalist no-chance' instead, just to avoid any confusion with the distinction this study draws between 'general probability' and 'single-case probability' or 'chance'. It examines what the physicalist no-chance interpretation says about the chance assertions in quantum mechanics and statistical mechanics. *Section 5.4* draws features from the BSA equipped with 'Bridgman's guide' and the physicalist no-chance interpretation and compiles the Humean propensity analysis of chance. Finally, it suggests that the Humean propensity proposal captures many of the intuitions usually assigned to the concept of chance.

5.1 Lewis' Subjectivist's Guide to Chance

Lewis (1986a) account of chance is constituted by two components: the BSA, his metaphysical theory of chance and the Principle Principle (PP), a principle specifying a relation between chance and rational credence. Due to certain inconsistencies between the BSA and the applications of the PP–the problem of 'undermining futuresdiscussed later on–Lewis (1994) substitutes the PP with his 'New Principle' (NP). He considers that especially the PP, and to some good extent the NP, capture all of our pre-theoretical intuitions about the concept of chance. Both versions of

Lewis' account seem to have the same aim: to reduce the concept of (BSA) chance in terms of the PP and, in the revised version, in terms of the NP. The idea is roughly the following: In Lewis' account of chance (BSA + PP), the BSA provides a definition of some probabilistic function, 'BSA chance'. Lewis' claim is that BSA chance is the *chance* function and not some other probability function i.e. the frequencies, because the BSA chance is the right plug-in for the PP. Similarly, in the revised version (BSA + NP) the claim is that BSA chance is the *chance* function because it is the right plug-in for the NP. In this section I describe the components of Lewis' accounts of chance (BSA, PP, NP) and note some problems that arise by attempting to analyse BSA chance in terms of these principles. Also, I propose an alternative way to conceptualise the BSA used to develop the Humean Propensity account in Sec.4.

5.1.1 Best System Analysis

According to the BSA, the laws of nature are the theorems entailed by the axioms that provide the best systematisation of the world. The chances are the probabilities entailed by the best system. More precisely, Lewis builds the BSA upon the metaphysical doctrine of Humean Supervenience (henceforth HS). HS holds that: (1) all the fundamental natural properties of the world are categorical (non-modal) and (2) all truths supervene on the patterns of instantiations of fundamental properties. The laws of nature and the chances supervene on the distribution of categorical properties throughout the entire space-time; the Humean Mosaic. Different systems competing for the title of best provide different systematisations of the Humean Mosaic. How good is the systematisation that each provides is determined by three theoretical virtues: 'simplicity', 'strength' and, when chances enter the picture, 'fit'. The general idea is that the arrangement of properties of the Humean Mosaic provides the candidate systems competing for the title of best, where the theoretical virtues and the balance between them determine which system prevails. To illustrate how this works let me briefly describe the BSA of laws of nature first without chances, and then with chances.

5.1.1.1 BSA of laws without chances

Before chances enter the picture, the Humean Mosaic provides the candidate systems that enter the competition and the best balance between the theoretical virtues of simplicity and strength determines the systematisation that comes out as best. The theorems of the best system are the laws of nature. Lewis (1994, p.478) summarises the general idea as follows:

> Take all deductive systems whose theorems are true. Some are simpler, better systematized than others. Some are stronger, more informative, than others. These virtues compete: an uninformative system can be very simple, an unsystematized compendium of miscellaneous information can be very informative. The best system

> is the one that strikes as good a balance as truth will allow between simplicity and strength [...] A regularity is a law iff it is a theorem of the best system.

So, only true deductive systems are allowed to enter the competition (the Humean Mosaic determines the systems whose theorems are true). The rules of the competition state that the best of these true deductive systems is the one that achieves the optimal balance between simplicity and strength. The theorems of the best system are the laws of nature. These laws of nature have the form of universal generalisations. They are universal generalisation that acquire the status of a law by being part of some integrated system of truths that combine simplicity and strength in the best possible way (Lewis, 1986a, pp.121-122).[1]

Consider first the virtues of simplicity and strength whose best balance determines the best of the true deductive systems that enter the competition and whose theorems are the laws of nature. *Simplicity* is characterised in terms of the complexity of the logical and the mathematical form of the system's axioms and the number of axioms the system has; the less complex the logical and the mathematical form of the axioms and the fewer the axioms, the simpler the system (Lewis, 1994, p.479). Regarding *strength*, Lewis (1994, p.480) states that a system is strong in case it says "what will happen or what the chances will be when certain situations of certain kind arise". To avoid the reference to chance for now, strength can be characterised as a measure of how informative the system is about the Humean Mosaic as a whole; the broader the scope of a system, the higher its strength. The idea is that typically, the theoretical virtues of simplicity and strength trade off. A system can become stronger (more informative) by adding further axioms or by making the mathematical and the logical form of its axioms more complex but this will come at the cost of simplicity. In the same manner, a system can become simpler by removing axioms or by making the mathematical and logical form of its axioms less complex, but this will come at the cost of strength.

Lewis' characterisation of simplicity and strength makes the concepts dependent on the language of each system; simplicity and strength are vocabulary-relative. He therefore requires for the systems that enter the competition to be first translated into a common language L where

[1]Lewis' regularity theory of laws of nature is both 'collective' and 'selective' (Lewis, 1986a, p.123). It is collective because the regularities that count as laws do not earn their lawful status in isolation. Rather, their status as laws is gained because they appear either as axioms or as theorems in the system that provides the best systematisation of the world. The theory is also selective in the sense that it does not say that *any* regularity counts as a law. Some regularities are regarded as merely accidental e.g. regularities that if included as an axiom or a theorem to the best system they would make it more complex without adding sufficient strength that counterweights the increase of complexity. Also, only collections of truths can be laws of nature; that is, only generalisations within the system count as laws of nature yet individual truths may impact which systematisation balances the theoretical virtues in the best possible way. As Lewis (1986a, p.124) puts this: "I do not say that the competing integrated systems of truths are to consist entirely of regularities; however, only the regularities in the best system are to be laws. It is open that the best system might include truths about particular places or things, in which case there might be laws about these particulars". For instance, in Lewis' account an individual truth (e.g. facts about the Big Bang) would not count as a law of nature as it cannot be generalised. However, individual truths do count towards the complexity of the system. Consequently, they are to be taken into account in the evaluation of the balance the system scores between simplicity and strength (and when chances enter the picture, fit).

all predicates denote 'perfectly natural' properties.[2] Thus, before chances enter the BSA, the best system is the system expressible in L that achieves the optimal balance between simplicity and strength as far as truth allows. The theorems of the best system are the laws of nature.

5.1.1.2 BSA of laws and chances

When chances enter the BSA, the conditions for deductive systems to enter the competitions and the rules of the competition change. Regarding the former Lewis (1986a, p.480) writes:

> We hold a competition of deductive systems, as before; but we impose less stringent requirements of eligibility to enter the competition, and we change the terms on which candidate systems compete. We no longer require a candidate system to be entirely true, still less do we require that it never had any chance of being false. Instead, we only require that a candidate system be true in what it says about history; we leave it open, for now, whether it also is true in what it says about chances. We also impose a requirement of coherence: each candidate system must imply that the chances are such as to give that very system no chance at any time of being false.

Thus, when chances enter the BSA the eligibility conditions for candidate systems that allow to enter the competition change in two respects: (1) they are no longer required to be true in their entirety but only in what they say about the history (the probability function is not interpreted yet) and (2) they "must imply that the chances are such as to give that very system no chance at any time of being false" (Lewis, 1986a, p.480). That is, chances enter the picture by allowing that systems competing for the title of best to include statements that specify the chances of events, in addition to statements regarding what happens in the history of the world. The systems that enter the competition may include a so far uninterpreted function P_t assigning chances at times. More precisely, a so far uninterpreted $P_t(s)$ mapping proposition and time pairs (s,t) onto real values in the $[0,1]$ interval (Loewer, 2004). The rough idea is that if the history of the world includes many 'indeterministic-like' events, a candidate system could gain strength with little cost on simplicity by having a probability function that assigns 'chances' to these events (Fenton-Glynn, 2014). The rules of the competition also change:

> Once we have our competing systems, they vary in simplicity and in strength, as before. But also they vary in what I shall call fit: a system fits a world to the extent that the history of that world is a comparatively probable history according to that

[2]More precisely, he requires that all competing systems must be first translated into language L, whose atomic predicates express only fundamental properties–'perfectly natural properties'–and spatio-temporal predicates such that the truths of L designate the geometrical structure of space-time as well as the 'perfectly natural properties' instantiated in each point. Accordingly, all the truths of the world supervene on the totality of truths expressible in L where a deductive system in L is a set of sentences in L. All deductive systems in L whose theorems are true are considered as candidates competing for the title of best (Loewer, 2004).

> system [...] The best system will be the winner, now, in a three-way balance between simplicity, strength, and fit (Lewis, 1986a, p.480).

That is, the systems that enter the competition are now judged, in addition to the theoretical virtues of simplicity and strength also in terms of fit. Regarding *fit* Lewis (1994, p.480) says that a systematisation fits the world in case the probability function that the system endows assigns high probability to the actual history of the world. The higher the probability the system assigns to the actual course of history, the better its fit.[3]

History determines the fit of each of the competing systems. Add the virtues of simplicity and strength and the system that scores the best balance between the three is the best system. The idea is that the strength and the fit of a system can be improved at the cost of simplicity. Similarly, a system may increase its fit and simplicity at the expense of strength e.g. by assigning chances to events rather than specifying the actual outcome of each. The probability function that appears as an axiom or a theorem in the system that combines simplicity strength and fit in the best possible way is a probabilistic law; it is the *chance function* of the Humean Mosaic:

> As before, the laws are the generalizations that appear as axioms or theorems in the best system; further, the true chances are the chances as they are according to the best system. So it turns out that the best system is true in its entirety— true in what it says about chances, as well as in what it says about history. So the laws of chance, as well as other laws, turn out to be true; and further, to have had no chance at any time of being false. We have our Humean supervenience of chances and of laws; because history selects the candidate systems, history determines how well each one fits, and our standards of selection do the rest (Lewis, 1986a, p.480).

In his 'Humean Supervenience Debugged' Lewis (1994, p.480) summarises the BSA of laws and chances:

> The best system is the system that gets the best balance of all three. As before, the laws are those regularities that are theorems of the best system. But now some of the laws are probabilistic. So now we can analyse chance: *the chances are what the probabilistic laws of the best system say they are*" [My emphasis].

Thus, the *best* system is *true* in its entirety; it is true in regards to what it says about both the history and the chances. In general, *if* there is a unique best systematisation of the Humean

[3]If the chance events are infinitely many, this characterisation of fit does not apply. (Elga, 2004) suggests a characterisation of fit as 'typicality' that extends to infinite cases. He takes the formalisation of the notion of a world being typical with respect to a probability function from Gaifman and Snir (1982) saying that: Given a world w, a probability function P, and a set T of test propositions, w is typical with respect to P and T if and only if P assigns non-zero probability to every test proposition that is true at w . He suggests that test propositions are those expressible in a certain first-order language i.e. the language L of fundamental physics. Schwarz (2018) suggests that 'fit' is to be measured by 'chi-squared' tests statisticians employ to test relationships between categorical variables.

Mosaic of our world, the BSA will deliver a determinate collection of true generalisations. The theorems or axioms of the best systematisation are the laws of nature and the chances entailed by the probabilistic laws (if there are any) are the BSA chances. A probability law (or a law of chance) is a law of nature for the same reasons that all other regularities acquire the status of a law; it is part of some integrated system of *truths* that combine simplicity, strength and fit in the best possible way (Lewis, 1986a, pp.121-122).

These probability laws have the form of 'history-to-chance conditionals': 'if H is the actual history up to and including time t, the chance at t of A at t_1 is x'. That is, the probability laws pertain to a later state of the world given an earlier state and the chances they entail are conditional on an earlier state of the world evolving over time by conditioning on the history. The BSA chances are entailed by the probabilistic ('history-to-chance') laws.[4]

In a nutshell, when chances are added to the BSA, the best system is the one that achieves the optimal balance between simplicity, strength and fit and the probability function associated with that system is a probabilistic law of nature; it is the chance function of the Humean Mosaic.

5.1.1.3 Variations of the BSA

So far it has been assumed that there is a uniquely-best systematisation of our world. This is not a trivial assumption. Lewis (1986a, p.124) considers this as the assumption that 'nature is kind'. If 'nature is kind' the arrangement of properties of the Humean mosaic together with the theoretical virtues and the balance thereof would determine a unique best systematisation of our Humean Mosaic. Yet, if 'nature is unkind', the arrangement of properties of the Humean mosaic together with the theoretical virtues and the balance thereof may lead to multiple best systems. That is, they would be multiple systematisations of the Humean Mosaic that scores (approximately) the same in terms of the balancing of the theoretical virtues. Lewis (1986a, p.124) initial idea is that "[i]f two or more systems are tied for best, then certainly any regularity that appears in all the tied systems should count as a law". That is, the laws of nature are the generalisations common to these tied for best systems. Later, Lewis (1994, p.479) suggests that if 'nature is unkind' and there is no good systematisation of the Mosaic, a Humean should say that there are no laws of nature at all.

[4]The canonical BSA considers this to hold as a metaphysical necessity. Lewis (1986a, p.112) writes: "[i]n saying what makes a certain proposition be the complete theory of chance for a world [...] I gave an explanation in terms of chance. Could these same propositions possibly be picked out in some other way, without mentioning chance?". Lewis (1986a, p.111-113) provisional answer is "most likely not", suggesting that one can avoid this problem by considering that "[p]erhaps all worlds are exactly alike in the dependence of chance on history. Then the complete theory of chance for every world, and all the conditionals that comprise it, are necessary. They are supervenient on particular fact in the trivial way that what is noncontingent is supervenient on anything [...] Chances are still contingent [...] because they depend on contingent historical propositions [...] and not also because they depend on a contingent theory of chance". In effect, Lewis' proposal to avoid providing an analysis of chance that does not mention 'chance' is to consider that the BSA holds out of metaphysical necessity; that is, it is a metaphysical necessity that the chances in a world are what the probability laws in that world say they are.

What implications the possibility that 'nature is unkind'–and that there is no uniquely best systematisation of our world–has for the BSA chances is a subtle question. There is an ongoing debate on what the possibility of tied systems implies for the BSA of chance. For instance, Beisbart (2014) argues that if there is no unique best system for our world then nothing deserves the name 'chance'. More precisely, he argues that if it is the case that the Humean Mosaic together with the theoretical virtues of simplicity, strength and fit do not single out a unique best system and consequently a unique corresponding probability function, then nothing counts as a probabilistic law–as the chance function of our world–and that this implies that nothing deserves the name of 'chance'. On the other hand, Dardashti et al. (2014) and Fenton-Glynn (2019) argue that the correct conclusion to be drawn in cases of ties is that any probabilities that the tied for best systems agree upon count as *imprecise* chances; that is, they correspond to the *sets* of probabilities entailed by these systems.[5]

This debate relates to the well-known problems with the BSA regarding: (1) the status of the theoretical virtues of simplicity and strength, and (2) whether a precise characterisation of them is possible. Regarding the status of these theoretical virtues, Lewis (1986a, p.124) observers that they seem to come from, or at least depend on, us: "they are those that guide us in assessing the credibility of rival hypotheses as to what the laws are. In a way, that makes lawhood depend on us-—a feature of the approach that I do not at all welcome!". In his 'Humean Supervenience Debugged' Lewis (1994, p.479) suggests that this problem need not to arise if 'nature is kind' to us:

> If nature is kind, the best system will be robustly best-so far ahead of its rivals that it will come out first under any standards of simplicity and strength and balance. We have no guarantee that nature is kind in this way, but no evidence that it isn't. It's a reasonable hope.[...] I suggest we not cross these bridges unless we come to them.

Even if we accept such an unpleasant consequence we wouldn't want these theoretical virtues to heavily depend on one's pre-theoretical intuitions. Rather, the aim would be to introduce some formal tools that would allow us to quantify these theoretical virtues in a more or less objective way. For example, consider the previous discussion about the possibilities of ties between different systematisations of our world. In order to evaluate whether certain systematisations of our world score better than others, or that they are (approximately) tied etc., one would need a formally precise characterisation of these theoretical virtues. This is a very difficult task. For instance, consider simplicity characterised in terms of the complexity of the logical and the mathematical form of the axioms of the system and how many axioms the system has. The number of the axioms of a system can be easily quantified by counting the axioms. This does not seem to be the case with the *complexity* of the logical and mathematical form of the axioms which would require one

[5]For a detailed discussion regarding the proposal that in case of a tie between systems the BSA chances are imprecise chances see Fenton-Glynn (2019, pp.21-32).

to precisely quantify the concept of complexity or 'complexity measure'. The problem is that there are a lot of different ways one can define complexity measures which may suggest that there is a degree of arbitrariness of what complexity measure is to be used in the characterisation of simplicity. To avoid this, one would need a consistent monotonic ranking of different complexity measures which would allow the use of only equivalent complexity measures in characterising simplicity. It remains an open question whether such a consistent and monotonic ranking of different complexity measures can be provided.[6]

On the other hand, Schwarz (2014) surmises that how exactly these details are to be spelled out is not crucial to the best system approach; the impreciseness of these theoretical virtues is a bullet that advocates of the BSA seem willing to bite. Notwithstanding, the theoretical virtues appear to play an important conceptual role in the BSA; they determine which, if any, systematisation of a world comes out as the best and to that extend the laws of nature and the chances, if any, the laws entail.

Another subtle debate concerns the compatibility (or not) of the BSA of chance and of determinism. For instance, Lewis' canonical BSA of chance is an incompatibilist position; that is, he thinks that if determinism is true then there can be no non-trivial–with values other than 1 and 0–chance assertions. He writes: "if the chance is zero or one, [...] then it cannot also be 50%. To the question of how chance can be reconciled with determinism, or to the question of how disparate chances can be reconciled with one another, my answer is: it cannot be done" (Lewis, 1986a, p.118). To this end, Lewis' formulation of the BSA targets fundamental dynamical chances existing only to non-deterministic worlds; that is, probability functions that pertain to a later state of the world given an earlier state. These fundamental dynamical chances are always conditional on an earlier state of the world and evolve over time by conditioning on the history; they have the form of 'history to chance conditionals'.[7]

Yet, many authors have argued that probabilities can appear in the best system in different ways.[8] Certain modifications of the BSA allow for the best system to include a non-dynamical probability distribution over initial conditions of the universe as it is the case with certain formulations of statistical mechanics and Bohmian mechanics (Hájek et al., 2011). Since non-dynamical probability distribution over initial conditions is compatible with determinism, one could extend the BSA to deterministic chances. For instance, Loewer (2004) argues that the BSA can be modified such that the single-case probabilities of statistical mechanics are BSA chance

[6]For a discussion on complexity measures that illustrate this point see Crutchfield and Wiesner (2010), and Ladyman et al. (2013).

[7]Even if we suppose that incompatibilism is to be considered as a condition of BSA, this does not necessarily impose a problem in extending Lewis' canonical BSA to phenomena of statistical mechanics e.g. coin tosses, spin of a roulette etc. The question of whether or not the dynamics of statistical mechanics are deterministic (or not) is yet to be conclusively settled. Norton for instance argues that statistical mechanics is indeterministic while Werndl makes a strong case that the jury is still out regarding its deterministic (or not) nature (Norton (2003), Norton (2008), Werndl (2016)).

[8]For such proposals see Loewer (2001), Loewer (2004), Loewer (2007), Loewer (2009), Hoefer (2007), Frigg and Hoefer (2010), Fenton-Glynn (2009), Fenton-Glynn (2019).

assertions even if we suppose that statistical mechanics are deterministic. His argument goes roughly like this: If one adds a probability distribution over the initial conditions of the universe to other laws it will result to a tremendous gain in terms of the system's strength which will outweigh the loss of simplicity caused by the addition of the probability distribution. By doing so, the laws of such system in conjunction with the probability distribution may indeed be the best systematisation of the Humean mosaic. And since probability distributions over initial conditions of the universe are compatible with deterministic dynamical laws, Loewer (2001, p.618) concludes that BSA is compatible with deterministic chances.

More precisely, Loewer (2009) uses the axiomatisation of statistical mechanics by Albert (2001) according to which statistical mechanics can be derived by three axioms: (1) the fundamental dynamics, (2) the past hypothesis (a proposition characterising the initial condition of the universe as constituting a low-entropy state) and, (3) the statistical postulate (a uniform probability distribution over the regions of micro-physical space associated with that low-entropy state). The claim is that these axioms entail the probabilities of statistical mechanics in, roughly, the following manner: by the past hypothesis, there is a region of microphysical space associated with the low-ethropy state of the universe. The fundamental dynamics lead to an increase of entropy until thermodynamical equilibrium is reached where the volume of microphysical space associated with the low-ethropy state of the universe relative to the total volume of that region is very high. As a result, by the statistical postulate, the uniform probability distribution over the entire region gives a very high probability that the universe follows that trajectory.[9] He concludes that the conjunction of these three axioms, what he calls 'the Mentaculus', derives the probabilities of statistical mechanics and many of the probabilities of the special sciences. Since this system is much stronger than one only consisting of the fundamental dynamical laws and it is not much more complicated (it only requires two further axioms), Loewer suggests that the Mentaculus may very well be the best system of our world.

Various objections have been raised against his argument. First, in order for the Mentaculus to be even considered as a candidate competing for the title of best, the initial conception of Lewis' BSA needs modification. Schaffer (2007, p.130) notes that the system contains predicates like 'low entropy' that do not correspond to 'perfectly natural' properties. Yet, the BSA requires that before any system enters the competition, it must be first translated into a common language whose predicate denotes 'perfectly natural' properties.

On the other hand, Fenton-Glynn (2019, pp.4-5) drawing on previous remarks from Lewis (1983, p.368) that the concept of naturalness comes into degrees, suggests the following modification of the BSA that does not deviate much from Lewis' initial conception: the *naturalness* of the predicates that a system employs may be considered as another theoretical virtue to be weighted with simplicity, strength and fit. If a system of axioms employs a 'not too unnatural' predicate i.e. 'low entropy' in the Mentaculus, and by doing so achieves greater simplicity, strength and fit,

[9]For a detailed discussion see Fenton-Glynn (2019, pp.3-10).

then it counts as a candidate competing for the title of best. The impreciseness of the concept of 'naturalness of predicates' does not seem to impose a great threat to Fenton-Glynn's suggestion. At the end of the day, the characterisation of the virtues of simplicity and strength do not fair much better. As long as advocates of the BSA are willing to accept the impreciseness of simplicity etc. they might be willing to also accept the impreciseness of 'naturalness of predicates'.

The second problem that Schaffer (2007, p.132) raises against Loewer's argument is that "given the deterministic dynamics and the initial conditions, the system is already maximally strong. Any extra 'laws' projecting micro-posterior chances prove needless". That is, if the fundamental dynamics of the system are deterministic then the system is already maximally strong such that the addition of the past hypothesis and of the statistical postulate would make the system more complex (less simple) without adding extra strength.

The third and perhaps most important problem with Loewer's argument evolves around the concept of 'entropy of the universe' that the past hypothesis refers to. There is an open debate in cosmology whether the 'entropy of the universe' is a well-defined concept. Many authors have argued that when the concept is defined by actual theories in cosmology it does so based upon arbitrary conventions and in some cases it cannot be defined at all (Earman (2006), Curiel (2015), Gryb (2020)). If this is the case, the 'entropy of the universe' may not even count as a 'not too unnatural predicate'.

Another subtle issue–related to the compatibility (or not) of BSA with determinism–is how far from fundamental physics the BSA of chance can be extended.[10] Frigg and Hoefer (2010) argue that this depends on one's view regarding: (1) the 'constituents' of the Humean mosaic and (2) their stance on ontological reduction or ontological pluralism. In brief, ontological reductionism is the thesis that the Humean mosaic consists of elementary particles and their trajectories, whereas ontological pluralism says that elementary particles are as real as computers, pencils, cats etc.

Regarding the compatibility of BSA and determinism, the Humean propensity account that this chapter aims to develop it is compatible with both dynamical and non-dynamical chances. Regarding ontological reductionism or pluralism, the Humean propensity account is, I believe, compatible with both. The proposal only requires that Humean Mosaic does not include 'irreducible modalities', 'powers', 'irreducible propensities', 'necessary connections', 'natural necessities', 'potentialities' etc.

5.1.1.4 A basic problem?

Let us return to the main question this chapter examines; that is, 'what could chance assertions like 'Ch(heads)= 0.5' possibly mean given one's commitment that they are objective descriptions of the world?'. The BSA gives the following answer: 'Ch(heads) = 0.5' means that the probability

[10] For an argument that the BSA analysis of chance can be extended to the probabilities in special sciences even if their laws are deterministic see Fenton-Glynn (2009).

laws of the physical theory that achieves the optimal balance between the theoretical virtues of simplicity, strength and fit (and maybe some other virtues like naturalness of predicates) assigns the value of 0.5 to this coin landing heads. That is, ‘the chance of this coin landing heads is 0.5’ is true *if and only if* 0.5 is the probability for this coin landing heads entailed by the set of axioms that best systematises of the Humean Mosaic. More generally, according to the BSA, the chance of an event is defined as whatever the probability laws of the best system of our world–the chance function of our world–assign(s) to the event. If the probability laws of our world–the chance function of our world–assign(s) probability x to an event, then the chance of that event is x. Likewise, if the chance of an event is x, then x is the value that the probability laws of our world–the chance function of our world–assign(s) to that event.

On the other hand, Loewer (2001, p.1112) observes that Lewis (1994, p.484) seems to suggest the BSA chances involve symmetries and frequencies when he writes that “I can see, dimly but well enough, how knowledge of frequencies and symmetries and best systems could constrain rational credence”. Nevertheless, it is not a straightforward consequence of the BSA that its chance assertions can be justified on the basis of symmetries or on the basis of frequencies. That is, the assertion ‘Ch(heads)= 0.5’ understood in terms of BSA does not mean that the coin-toss set up is symmetric nor that the relative frequency of heads is 0.5. It merely means that the probability laws–the probability function that appears as a theorem or as an axiom in the systematisation that achieves the optimal balance between the theoretical virtues of simplicity, strength, fit, naturalness of predicates etc.–assign(s) the value of 0.5 to the particular coin-toss landing heads. This is not to deny that the chances may be close to frequencies since the best system is contacted to fit the actual history of the world and thus the actual frequencies. Rather, the claim is that it is not a straightforward consequence of the BSA that the chances can be understood in terms of frequencies.[11]

There seems to be a basic reason to question the degree in which the BSA, just by itself, provides an informative analysis of chance. If the chances in a world are defined by saying that they are whatever the probability laws of the world say they are–or, what the chance function of that world says it is–and explicitly require that what they say is true then, one would not be able to understand what the probability laws were unless they have already understood what the chances are. Similarly, one would not be able to understand what the chances are unless they already knew what the probability laws of the world were–unless they already knew what the chance function of the world was.

[11]Lewis (1986a, pp.128-129) writes on this: We will tend, *ceteris paribus*, to get the proper agreement between frequencies and uniform chances, because that agreement is conducive to fit. But we leave it open that frequencies may chance to differ from the uniform chances, since *ceteris* may not be *paribus* and the chances are under pressure not only to fit the frequencies but also to fit into a simple and strong system.

5.1.1.5 A proposal

We can possibly avoid this problem by not requiring for the probability laws in a world to be true in terms of what they say about the chances in that world. Accordingly, we can consider that the probability laws–the chance function of our world–'fix the reference' of chance in our world without revealing the nature of the quantity they are referring to. This would allow to use the analysis of chance provided by the BSA to say something non-trivial about chance.[12] In a nutshell, the proposal is to consider that the BSA *partially* defines the concept of chance in the world; that is, the *contingent* probability laws of our world 'fix the reference' of the concept of chance in our world without revealing the nature of the quantity or magnitude the concept refers to.[13]

5.1.2 Subjectivist Guide(s)

As mentioned, in Lewis' account of chance (BSA + PP), the BSA gives us a definition of some probabilistic function 'BSA chance'. Lewis proposes that BSA chance is the *chance* function because the BSA chance is the right plug-in for the PP; the principle that Lewis claims to be capturing all of our pre-theoretical intuitions about chance. Similarly, in the revised version of his account (BSA + NP), Lewis' claim is that BSA chance is the *chance* function because it is the right plug-in for the NP; the principle that captures–to some good approximation–the pre-theoretical intuitions about chance that the PP captures. In this section I describe the PP and the NP and note some problems that arise by attempting to analyse the BSA chance in terms of these principles.

5.1.2.1 Principal Principle

Lewis (1986a) in his seminal work 'A Subjectivist's Guide to Objective Chance' begins by giving undefended answers to his own questionnaire suggesting that if one finds them "obviously right [one] will be willing to take them as evidence for what follows" (Lewis, 1986a, p.86). His claim is that the set of intuitions that motivate his answers to his questionnaire captures all of our pre-theoretical intuitions about chance. The full questionnaire and the answers follow:

[12] Presumably Lewis wants to do so by defining chance in terms of his 'Principal Principle' and in the revised version in terms of his 'New Principle' (these are discussed in the next section)

[13] This proposal, while its motivations differ, is similar to the version of the BSA that Lewis (1986a, pp.127-128) suggests in the postscripts of his 'Subjectivist's Guide to Objective Chance' that takes chances as *given* and derives the probability laws: "The laws—-laws of chance, and other laws besides—-supervene now on the pattern of particular chances. If the chances in turn somehow supervene on history, then we have Humean supervenience of the laws as well; if not, not. The corrected theory of lawhood starts with the chances. It does nothing to explain them." See Lewis (1994, p.127) for the reasons he suggest this version of the BSA (the main reason is the inconsistencies between the BSA of laws and chances together and certain applications of the Principal Principle discussed in the next section). In Lewis (1994), he substitutes the Principal Principle with the New Principle that avoids these inconsistencies and he adopts the BSA of laws and chances together.

> [Question 1:] A certain coin is scheduled to be tossed at noon today. You are sure that this chosen coin is fair: it has a 50% chance of falling heads and a 50% chance of falling tails. You have no other relevant information. Consider the proposition that the coin tossed at noon today falls heads. To what degree would you now believe that proposition? Answer. 50%, of course
>
> [Question 2:] As before, except that now it is afternoon and you have evidence that became available after the coin was tossed at noon [...] You remain as sure as ever that the chance of heads, just before noon, was 50%. To what degree should you believe that the coin tossed at noon fell heads? Answer. Not 50%, but something not far short of 100%.
>
> [Question 3:] As before, except that you have plenty of seemingly relevant evidence tending to lead you to expect that the coin will fall heads. This coin is known to have a displaced center of mass, it has been tossed 100 times before with 86 heads, and many duplicates of it have been tossed thousands of times with about 90% heads. Yet you remain quite sure, despite all this evidence, that the chance of heads this time is 50%. To what degree should you believe the proposition that the coin falls heads this time? Answer. Still 50% .
>
> [Question 4:] You have no inadmissible evidence; if you have any relevant admissible evidence, it already has had its proper effect on your credence about the chance of heads. But this time, suppose you are not sure that the coin is fair. You divide your belief among three alternative hypotheses about the chance of heads, as follows. You believe to degree 27% that the chance of heads is 50%. You believe to degree 22% that the chance of heads is 35%. You believe to degree 51% that the chance of heads is 80%. Then to what degree should you believe that the coin falls heads? Answer. (27% × 50%) + (22% × 35%) + (51% × 80%); that is, 62% (Lewis, 1980, pp.84-86 4).

Lewis (1986a, p.86) introduces the Principal Principle (henceforth PP) the principle that "captures all we know about chance", aiming to make the pre-theoretical intuitions that support his answers to his questionnaire precise. In effect, the PP specifies a relation between chances and rational credence. In its simplest version, the PP says that one, if rational, should set her credence equal to the chances (BSA chances) given she has no inadmissible information. More precisely, let '$C(.)$' be a rational initial credence function, 'A' any proposition that the agent may have beliefs about, 'K' any admissible information and '$Ch(A) = x$' a proposition stating that the chance of A is equal to x, then the PP states:

(PP): $C(A|`Ch(A) = x'\&K) = x$.

With the PP in place, chance is defined as that function that plays the PP role. More precisely, as discussed, in Lewis account of chance (BSA + PP), BSA defines a probabilistic function 'BSA chance'. Lewis argues that the BSA chance is actually the chance function because it is the right plug-in for the PP. The PP in its turn, Lewis argues, captures all of our intuitions about chance. In this sense, the BSA together with the PP completes the initial version of Lewis account of chance; that is, with the BSA and PP in place, the BSA chance is reduced in terms of the PP (sic.).

5.1.2.2 Problems of reducing chance to the PP

Let's now turn on some problems with the attempt to analyse chance in terms of the PP. The main issue is whether the concept of *rational initial credence* can be defined without making essential reference to the concept of *chance*. As Lewis observes:

> Doubtless it has crossed your mind that [the PP] has at least the form of an analysis of chance. But you may well doubt that it is informative as an analysis [...] Not that it has to be informative as an analysis to be informative. [The problem is] the allusion in the analysans to reasonable initial credence functions. [...] [C]ould we possibly get any independent grasp on this concept, otherwise than by way of the concept of chance itself? [...] my provisional answer is: most likely not, but it would be worth trying [...] What might we try? A reasonable initial credence function ought to (1) obey the laws of mathematical probability theory [the probabilism constrain on rational credence]; (2) avoid dogmatism, at least by never assigning zero credence to possible propositions and perhaps also by never assigning infinitesimal credence to certain kinds of possible propositions [regularity constrain on rational credence]; (3) make it possible to learn from experience by having a built-in bias in favour of worlds where the future in some sense resembles the past; and perhaps (4) obey certain carefully restricted principles of indifference, thereby respecting certain symmetries. Of these, criteria (1)–(3) are all very well, but surely not yet strong enough [...] It is less clear what (4) might be able to do for us. Mostly that is because (4) is less clear simpliciter, in view of the fact that it is not possible to obey too many different restricted principles of indifference at once and it is hard to give good reasons to prefer some over their competitors. It also remains possible, of course, that some criterion of reasonableness along different lines than any I have mentioned would do the trick (Lewis, 1986a, pp.110-112).

In effect, the problem that Lewis notes is this: since the aim is to analyse chance in terms of the PP and since the PP refers to a 'rational initial credence function', a well-defined conception of the 'rational initial credence function' that makes no essential reference to chance is needed. Otherwise, the attempt to analyse chance in terms of the PP will lead to circularities; the concept of rational initial credence function would make essential reference to the chance function and vice versa. Lewis (1986a, p.111) attempts to avoid this problem by suggesting the aforementioned four conditions that the initial rational credence function in the PP should satisfy where these make no reference to chance. But, at least for Lewis, they are not specific enough to pick out a unique concept of rational initial credence function. If we therefore follow Lewis' conception of 'reasonable initial credence function' in the PP, it becomes unclear if the concept can be well-defined without making essential reference to chance.[14] This problem is expressed fittingly by Szabó (2010, p.4) when he states that the "subjectivist's guide to objective chance would, first of all, require a guide to the subjectivist's [rational] credence".[15]

To illustrate the problem that this imposes to Lewis account (BSA + PP), recall that the PP says that the formula $C(A|`Ch(A) = x\text{'}\&K) = x$ only holds when information K is admissible. This is not a trivial condition especially if the aim of the PP is to help understand the concept of chance. As Lewis (1986a, p.93) notes "[t]he power of the Principal Principle [that tells us all we know about chance] depends entirely on how much is admissible. If nothing is admissible it is vacuous. If everything is admissible it is inconsistent".

Lewis provides two characterisations of admissibility in the PP. First, Lewis (1986a, p.93) provisional characterisation of the concept of admissibility says that: "admissible propositions are the sort of information whose impact on credence about outcomes comes entirely by way of credence about the chances of those outcomes". Maybe because Lewis thought his initial characterisation of admissibility was too vague he does not consider it as a definition of admissibility. He writes: "I have no definition of admissibility to offer, but must be content to suggest sufficient (or almost sufficient) conditions for admissibility" (Lewis, 1986a, p.93). He suggests that two types of information are admissible: (1) Historical information: "[i]f a proposition is entirely about matters of particular fact at times no later than t, then as a rule that proposition is admissible at t" (Lewis, 1986a, p.93). (2) Information about chance itself: "besides historical information, there is at least one other sort of admissible information: hypothetical information about chance itself [...] These admissible conditionals are propositions about how chance depends (or fails to depend) on history" (Lewis, 1986a, p.94).

Observe that both characterisations presuppose the concept of chance. The first characterisation does so: "admissible propositions are the sort of information whose impact on credence about outcomes comes entirely by way of credence about the *chances* of those outcomes" (Lewis, 1986a, p.93) [My emphasis]. So does the second of the (almost) sufficient conditions in the revised characterisation of admissibility: "besides historical information, there is at least one other sort of admissible information: hypothetical information about *chance itself* [...] These admissible conditionals are propositions about how *chance* depends (or fails to depend) on history" (Lewis, 1986a, p.94) [My emphasis].

The observation that the concept of admissibility in the PP presupposes that of chance imposes the following problem to Lewis' overall account of chance (BSA + PP): recall that the

[14] Maybe it is reasons like this that led Strevens (1999) to argue that a justification of the PP is no more possible than the justification of induction.

[15] Still, suppose that what the PP says is true. This would have the following strange implication: given that rational credence and chance are different things referring to different sort of phenomena then for the PP to be true either there must be a strict interaction between chance and rational credence or there must be something else in the *external world* that connects them.

BSA gives us a definition of a probabilistic function 'BSA chance'. But the important question is whether the BSA chance corresponds to our pre-theoretical intuitions of chance. Or is it simply a different probabilistic function (i.e. the frequencies etc.)? In Lewis' account (BSA + PP), the claim is that BSA chance is the *chance* function and not some other probability function because the BSA chance is the right plug-in for the PP. To examine whether BSA chance is indeed the right plug-in for the PP the following needs to be checked: Should a rational agent put her credence in A conditional on 'BSA chance(A) = x' & K to be x, when K is admissible? But, in order to determine if this holds, we firstly need to know which K is admissible. Nevertheless, it appears that we cannot do so unless an interpretation of chance is already in place. Otherwise, too many probability functions other than the BSA chance function could be the right plug-in for the PP i.e. the frequencies etc.

Put another way, in order for a rational agent to apply the PP she needs to be able to assess whether an information K is admissible relative to 'BSA chance(A) = x'. To do so, she needs to know what information counts as admissible. Yet, to know what counts as admissible she needs to have a concept of chance already in place, independent of the PP. Otherwise, it would have been impossible to know if K is admissible (or not) relative to 'BSA chance(A) = x' and in turn to apply the PP (or not).[16]

The observation that the concept of admissibility in the PP presupposes an interpretation of the concept of chance raises concerns regarding the tenability of the attempt to reductively analyse the concept of chance in terms of the PP, at least if one aims for a non-circular analysis of chance.

5.1.2.3 Undermining futures

Let's now turn to the problem that Lewis (1994) considers fatal for the reduction of BSA chance in terms of the PP and thus suggests replacing the PP with the NP; that is, certain applications of the PP are inconsistent with at least the canonical BSA of chance. Lewis calls this 'the problem of 'undermining futures'. Here is a brief description of the problem: Chances supervene on the Humean Mosaic if the facts about chances are entirely determined by non-probabilistic matters of fact. The matters of facts that determine the chance of any given event inhabit the past, the present, and the future; the chance of an event supervenes globally on the Humean Mosaic of particular matters of fact. The PP is inconsistent with any Humean Supervenience account of chance that allows for 'undermining futures'; that is, futures whose chance-making distribution determines a set of chances different from what the true chances actually are. The BSA has this feature:

Consider a theory of chance T understood in terms of the canonical conception of the BSA; that is, as the set of all true history-to-chance conditionals. Let H_{tw} be the complete history of the

[16]Some proposals seem to embrace this consequence. Loewer (2004) suggests to 'build in' the PP in the BSA of chance and Frigg and Hoefer (2010) argue that the PP is to be considered part of the definition of the concept of BSA chance.

world w up to an including time t. Let w be a world with an actual future in which $Ch(A)$ is 0.6. Let T be a theory of chance for w at time t and let A be some arbitrary proposition. Also, let G be some proposition about a future whose chance distribution says that the $Ch(A)$ is 0.4 and the chance of this future G according to T in w at t_1 is 0.3. Since according to T the $Ch(G) = 0.3$, then by applying the PP a rational agent should set her credence equal to 0.3; that is $C(G|HT) = 0.3$. Yet, G is incompatible with T because it assigns a value to $Ch(A)$ that is different from its actual value. Since HT implies that G is false a rational agent should set her credence to that falsehood to 0 ($C(G|HT) = 0$). Thus, the undermining future G which is a consequence of the BSA results to inconsistencies with these applications of the PP.

In effect, this problem appears because the second of the (almost) sufficient conditions of admissibility that Lewis suggests states that information about the chance itself is admissible. Yet, in analysing chance in terms of the BSA, information about the chance itself becomes inadmissible since it reveals information about the future–the matters of fact that determine the chance of an event supervene on past, present, and future matters of facts. Put another way, one can apply the PP if she has information about the chance itself yet not inadmissible information. But when the chances supervene on the Humean Mosaic as in the BSA, this can never be the case. The information about the chance itself is inadmissible, it reveals information about the future.[17]

5.1.2.4 New Principle

Lewis (1994) and Hall (1994) proposal to solve this problem is by substituting the PP with the New Principle (henceforth NP) which is compatible with the undermining futures. The NP says: let $C(.|.)$ be some reasonable initial credence function, A any proposition that an agent may have beliefs about, H_{tw} the complete history of the world w up to an including time t, T_w the complete history of chance for world w and P_{tw} the chance distribution at time t in w.[18] The NP states:

(NP): $C(A|H_{tw}T_w) = P_{tw}(A|T_w)$.

The NP is consistent with the possibility of undermining futures and in consequence with the BSA. To illustrate this consider that G is an undermining future, since T_w is by definition inconsistent with G the NP says that both $C(A|H_{tw}T_w)$ and $P_{tw}(A|T_w)$ are equal to 0. To that extend the NP says that in the case of the undermining future G, $C(G|HT)$ should be set to 0.[19] The reduction of BSA chance works as before. The difference is that now BSA chance is reduced in terms of the NP rather than the PP. That is to say, in Lewis' revised account (BSA + NP) the

[17] One could avoid this problem by rendering information about the chance itself as inadmissible information. Lewis (1986a, p.132) thinks that the cost of doing so is excessive because: "in ordinary calculations with chances, it seems intuitively right to reply on this hypothetical information [...] So much as I would like to use the best-system approach in defence of Humean supervenience, I cannot support this way out of our difficulty".

[18] The problem raised against the PP that the the concept of chance makes essential reference to that of reasonable initial credence function and vice versa affects the NP as well.

[19] In situations where there are no undermining futures, the NP becomes equivalent with the PP; if $P(T) = 1$ then $P(A|T) = P(A)$.

BSA chance is the *chance* function and not some other probability function i.e. the frequencies because it is the right plug-in for the NP.

5.1.2.5 Problems of reducing chance to the NP

Regarding the concept of admissibility, Lewis does not–at least not explicitly–discuss the concept once he accepts the NP. On the other hand, Hall argues that the NP does not require an admissibility clause ((Hall, 1994, p.512), (Hall, 2004, p.100)).[20] If Hall's argument is correct then the reduction of the concept of chance–or, that of 'conditional probability of an event given a complete theory of chance'–in terms of the NP avoids the threat of circularity. The threat arises because the concept of admissibility in the PP presupposes the concept of chance. According to Hall, since the NP does not require an admissibility constrain, the reduction of the 'conditional probability of an event given a complete theory of chance' in terms of the NP does not face this issue. Nevertheless, amending the concept of admissibility from the NP is not entirely unproblematic. The conceptual importance of admissibility is pointed out by Lewis (1986a, p.93) in his discussion on the PP: "[t]he power of the Principal Principle depends entirely on how much is admissible. If nothing is admissible it is vacuous. If everything is admissible it is inconsistent".[21] Since the reason Lewis substitutes the PP with the NP was to avoid the inconsistency with the BSA, one would expect that the power of the NP would also depend on how much is admissible. This is important in so far as the idea is that the NP captures, at some good approximation, the set of intuitions that support Lewis' answers to his questionnaire which motivate the PP and which he considers to capture all of our pre-theoretical intuitions about chance.

Also, even after accepting the NP as the solution to the problem of undermining futures Lewis (1994, p.489) writes that "I still find the old one more intuitive [...] So I still say that the old Principle is "the key to our concept of chance". And for good reasons. As Haddock (2011) argues, this is a technical solution to the problem of undermining futures and it comes with a conceptual catch. The PP was aiming to capture the intuitions that chance, understood as a probability measure over single-case events, is the proper kind of quantity that should constrain rational credence. The NP no longer supports the same intuitions since the concept of *single-case probability* measure has been replaced by that of the *conditional probability of an event given a complete theory of chance*. It is not clear that the quantities that each principle aim to capture are the same. Haddock (2011, p.858) suggests that we have intuitive reasons to doubt that this is the case: "[i]f we think that A has some objective chance, then it seems natural to suppose that it is the quantity of $P(A)$, instead of $P(A|T)$, that is of interest [...] conditionalising on a theory of

[20] For an objection to Hall's argument see Strevens (1995, pp.557-559) argument that without 'admissibility' the NP results to inconsistencies; in certain applications the conditional version of the NP gives a different answer from the canonical unconditional (or absolute) version.

[21] Strevens (1995, p.552) observation that "admissibility (or rather, inadmissibility) seems to be a phenomenon that arises in every epistemic context, not just those involving probabilities" seems to back up the importance that Lewis, at least initially, assigns to the concept of admissibility.

chance shouldn't have any bearing on the objective chance of an event; it is simply a different quantity". That is, if the aim is to analyse single-case probability rather than whatever quantity the concept of conditional probability of an event given a complete theory of chance refers to, then it is questionable whether the NP can be of much help.

5.1.2.6 Subjectivist Guides to Chance: conclusion

To conclude the discussion on Lewisian 'Subjectivist's Guides to Chance', I have considered that Lewis (1986a) initial analysis of chance is constituted by two components, the BSA and the PP, whereas Lewis (1994) revised analysis is constituted by the BSA and the NP. I have also considered that both versions aim for a reductive analysis of the concept of BSA chance in terms of the PP and NP respectively. I have raised some problems with the reduction of the concept of BSA chance in terms of these principles. Regarding Lewis' initial account of chance (BSA + PP) it has been argued that the concept of 'admissibility' in the PP makes essential reference to the concept of chance and this makes the overall account circular. Regarding Lewis' revised account of chance (BSA + NP) it has been argued that the NP that drops the admissibility constrain is not entirely unproblematic; the concept of 'admissibility', as Lewis notes, plays an important conceptual role to help us understand the concept of chance. Also, it is pointed out that the set of intuitions about chance that motivate the NP are not very clear.

I do not claim that these suffice to refute either version of Lewis' accounts of chance. Neither it is to be denied that–at least the PP–captures some of our pre-theoretical intuitions about chance. The hope is that they are enough to encourage–especially those who aim for a non-circular analysis of chance–to try a different approach to BSA chances that does not make essential reference to the PP or the NP. Also, I have proposed to consider the BSA as a partial analysis of the concept of chance; that is, the BSA 'fixes the reference' of the concept of chance in our world without revealing the nature of the quantity (magnitude) the concept refers to.

5.2 Alternative 'guides' to BSA Chance Assertions

In the previous section I have proposed to consider the BSA as a partial analysis of the concept of chance. The basic idea of this is to consider that the BSA 'fixes the reference' of the concept of chance in our world without revealing the nature of the quantity (magnitude) the concept refers to. In this section I focus on two alternative ways to approaches to BSA chances. First, Schwarz's proposal that BSA chance assertions need no interpretation (I call this proposal 'no-interpretation guide'). Second, I describe 'Bridgman's guide' that I instead propose; its basic claim is that operationalist joints play an indispensable role in our conceptual structure and have a constitutive–not necessarily exhaustive–role to play in the meaning of any given concept.

5.2.1 'No-interpretation' Guide

Schwarz (2018) proposes that a no interpretation of chance assertions is needed. He suggests that scientific probabilistic assertions can remain uninterpreted in view of his *new* approach on probabilistic scientific theories whose aim is to 'capture noisy relationships in the world' and which are not meant to be true or false (Schwarz, 2018). The motivation for his proposal derives from his considerations regarding probabilistic modelling in the sciences viewed from the perspective of BSA. He writes:

> [A]llow our scientist to specify a probabilistic relationship between F and G, perhaps by adding a noise term to an algebraic equation [...] The point of the model is to capture the noisy, stochastic relationship between F and G. It is not to capture a crisp relationship between F, G, and third quantity P. *This is why we could not find a sensible answer when we asked what that quantity might be* [...] When a scientist puts forward a probabilistic model, she commits herself to the assumption that the model fares well, on balance, in terms of simplicity, strength, fit and other relevant virtues. But this is not the content of her model [...] In order to serve its purpose, it is enough that the model contains a probability function. The function does not need an interpretation (Schwarz, 2018, p.1203) [My emphasis].

He suggests that "the probability claims in scientific theories are not meant to be true or false, and thus do not need an interpretation" (Schwarz, 2018, p.1198). Attempting an interpretation of these probabilities is to mistakenly assume that probabilistic theories are in the business of making straightforward, categorical claims about the world concluding that the question of 'what chance statements mean?' should be rejected all together.

There are two problems with the 'No-interpretation guide'. The first problem is that it is not clear what question the no-interpretation proposal aims to address. If the question Schwarz's proposal is concerned with is 'what probabilistic modelling amounts to', I agree that no further interpretation is needed. Gyenis and Rédei (2014) and Humphreys et al. (2008) provide convincing arguments that probabilistic modelling is understood at least as well as scientific modelling in general is. One the other hand, Schwarz (2018, p.1198) talks specifically about probability *claims* and not just about probabilistic modelling. At least for the purpose of this study, 'what probabilistic modelling is?' is not the relevant question nor the aim of an interpretation of 'chance'. The task of providing an interpretation, in this case of the concept of chance, is considered as the task of providing an analysis of the common features assigned to probabilistic assertions we categorise as chances.[22]

[22]More precisely, chapter 1 suggests that the aim of an 'interpretation of probability'–in the particular case of 'chance'–is to provide an analysis of the common features shared by probabilistic assertions derived by *applying probability theory* to phenomena 'external' to mathematics; that is, by using probability theory to *model* physical phenomena. The probabilistic assertions derived by doing so are categorised into classes depending on common features we think they share. Each class is considered as a distinct concept of probability that requires a distinct

The second issue with Schwarz's proposal concerns the reasons behind his argument that an analysis of 'chance' is unnecessary: that is, his new approach to probabilistic scientific theories as capturing noisy relationships in the world, whose chance assertions are not meant to be true or false. Nevertheless, it is not clear how the 'no-interpretation' proposal allows one to take seriously what the chance assertions of our current best physical theories e.g. statistical mechanics and quantum mechanics, say about the world. Surely for one to take something seriously, they must know what that something means.[23]

Having said that, an insight of the 'no-interpretation' proposal that I adopt for the Humean propensity analysis of BSA chance is the explicit use of the condition 'if scientific theories are viewed in certain way' in the argument. It indicates that, at least when the concern is probabilities in the physical sciences, their analysis can be developed within one's greater view of scientific theories.

5.2.2 Bridgman's Guide

This section discusses some basic tenets of Bridgman's philosophy that 'Bridgman's guide' relies upon. As discussed (Chapter 1), the approach to physical theories this study adopts is that of the operationalism/convetionalism tradition in philosophy of science. Gillies (2000, p.4) characterises this as the thesis that every new theoretical concept must be given an operational definition *in terms of experimental procedures and concepts already defined*. The empirical laws lying behind these definitions must be established by observations before introducing the new concept. A more precise formulation of the operationalist/conventionalist approach is due to Hofer-Szabó (2017, pp.2-3):

> *A physical theory* can be reconstructed as a formal system plus a semantics connecting the formal system to the the world. The *formal system* consists of a formal language with some logical axioms and derivation rules, some mathematical and physical axioms. The *semantics* provides an interpretation of the formalism; it connects the formal system to reality [...] [The] 'semantics' means a down-to-earth physical interpretation of the formal system connection between the formal system [...] [and] the semantics is an indispensable of a physical theory [...] [such as] a formal system in itself is *not* yet a physical theory [Emphasis in the original].

Roughly, an operationalist/conventionalist considers physical theories to be expressed in terms of quantitative relations amongst parameters with terms such as 'length', 'probability', 'time', 'mass' etc. Importantly, she considers that the way these *quantity-terms* apply to concrete

interpretation. Following Ismael (2011), the common features we assigned to chances are: (1) they correspond to objective features of the world, (2) they do not come with an explicit reference class and as a consequence (3) their basic form is unconditional (or absolute).

[23] Sec. 5.5 argues that contra-Schwarz, there is a *sensible* answer to the question of what sort of quantity 'chance' refers to.

particulars depends on nontrivial choices regarding the way the relevant quantity-terms are measured (Tal, 2017). However, operationalism/conventionalism is, in principle, compatible with scientific objectivity. That is to say, there is no reason to exclude the possibility that the convention (or choice) regulating the use of a given quantity-term is such that the quantity-term happens to *refer* to the world 'out there'. Especially for Bridgman (1927, p.5), the father of operationalism, the *meaning* of any concept is nothing more than–it is synonymous with–the corresponding set of operations it is measured by. We do not need to endorse such a strong version of operationalism however, but solely that the corresponding sets of operations have a constitutive, yet not exhaustive, role to play in the meaning of the concept. In terms of 'chance', we consider that operational definitions have a *constitutive*–not necessarily exhaustive–role to play in the meaning of that quantity-term; that is, that the meaning of 'chance' has an operational element in its definition.

Operationalism/conventionalism has of course, as any other approach in philosophy of science, subtle problems.[24] The current trend is to regard it as an extreme or even 'outworn' position. This however does not mean that the core ideas of operationalism have nothing important to teach. Operational definitions are familiar to philosophy of science and their importance cannot be neglected. Consider for instance scientific assertions like 'the length of (A) = 4m', 'the mass of (A) = 4 kg' and 'time mark of (A) = 3s'. There is an (at least basic) understanding of what these statements mean because they are coordinated with physical reality through some (conventionally) accepted units of measurement where each of these units is defined operationally. For instance, 'length' has meter as its unit which is currently defined as a measure of the distance light travels within 1/299792458 of a second. 'Mass' has the kilogram, currently defined as a measure of a prototype platinum-iridium cylinder while 'time' has the second currently defined as 9192631770 oscillations of a caesium-133 atom. The claim is not that these operational definitions exhaust the meaning of 'mass', 'length' and 'time' but the fact that their meaning has an operational component is what perhaps makes the assertions containing these terms meaningful to all.

At the end of the day, the philosophical worries that led Bridgman to develop operationalism are still relevant. Reflecting on Einstein's special theory of relativity's conception of 'distant simultaneity', Bridgman notes the dangers arising when stepping into new theoretical domains with old concepts in an unreflective way. Einstein's theory has taught, Bridgman at least, that the question of whether two spatially separated events are simultaneous does not necessarily have a definite answer; that is, the meaning of 'distant simultaneity' was not fixed unless an operation for measuring it was specified.[25] Thinking in operationalist terms, Bridgman argues,

[24]These are beyond the scope of this thesis. For a comprehensive discussion of the problems of operationalism see (Gillies, 1972) and (Chang, 2010).

[25]For this Einstein suggests the conventional 'Principle of Standard Synchrony': Let two spatial locations A and B be fixed at some particular, yet arbitrary, inertial frame of reference. Let a light ray travelling in vacuum leave A at time t_1 where the time is measured by a clock at rest at location A and arrives at B coincidentally with the event C at location B. Let the light ray be instantaneously reflected back to location A and consider that it arrives there at t_2.

would have allowed one to recognise from the start that the meaning of 'distant simultaneity' was not fixed unless an operation for measuring it was specified. Remaining aware of the joints that operationalism provides in our conceptual framework will prevent us, he argues, from falling into such traps again (Bridgman, 1927, pp.10-24).

I believe the following remark by Bridgman (1927, p.24) captures the essence of what I shall call 'Bridgman's guide' to BSA chance assertions: "we must remain aware of these [*operationalist*] joints in our conceptual structure if we hope to render unnecessary the services of the unborn Einsteins". In Sec. 5.4 I suggest that deeming 'Bridgman's guide' as *indispensable* for the meaning of BSA chance assertions: (1) avoids the problems arising with the other two guides and (2) reveals that the truth makers of BSA chance assertions are the ordinary, already known and well-defined physical quantities of Szabó's 'no-chance' interpretation discussed in the next section.

5.3 Szabó's Physicalist 'No-Chance' Interpretation of Chance

In this section, I focus on Szabó's novel physicalist 'no-chance' interpretation of chance assertions he developed throughout his work (Szabó (2001b), Szabó (2007b), Szabó (2010)). I then provide a complementary reading of Szabó's 'No-chance' interpretation and Lewis' BSA of the concept of chance.

In rough terms, the main idea of Szabó's interpretation is that there is no such *property* of an event as its 'chance'. 'Chance' is a collective term, the meaning of which varies from context to context. It means, in the operationalist sense, different dimensionless valued–in the interval [0,1]– physical quantities characterising different particular situations. Szabó (2007b, p.3), reflecting on the–somewhat paradoxical–situation that despite that none of the 'standard' interpretations of probability provides a satisfactory answer to the fundamental question 'what is probability?' then: "How is it possible that physics and other empirical sciences apply a formal (mathematical) theory of probability, without noticing a problem arising from this unanswered fundamental question?". He suggests that the apparent paradox arises because there is no such *property* of an event as its chance (Szabó, 2007b, p.4).

Before considering the example that he illustrates his view with, note two things about the importance of the specific example he uses. *First*, it is a case where 'chance' is defined by direct operational procedures. That is to say, it shows that it is possible to directly measure the 'chance' of an event. *Second*, it is a paradigmatic case for conceptualising what the BSA analysis of chance may imply for chance assertions in empirical science and physics in particular. Here is the example of Szabó (2007b):

Let a gun be placed in such a way it can shoot uniformly into a square size a^2 on the wall. Inside the square there is a round target with radius R and in the middle of the target there is

The Principle of Standard Synchrony says that event C is simultaneous with the event at location A that occurred at time $(t_1 + t_2)/2$. This is equivalent to the claim that the one-way speed of the light ray is the same on the two segments of its back and forth between locations A and B (Janis, 2018).

an aired balloon with radius r (assume that the balloon is round). Suppose also that the uniform distribution of shots in the square is ensured by positioning the gun to a computer which applies a suitable ergodic transformation map.[26] The employment of the ergodic transformation map guarantees the uniform distribution of shots without any reference to 'probability-theoretic' considerations but only from the equations of elementary kinematics–this ensures a uniform distribution of the shots while avoiding the threat of circularity and the reference to some sort of symmetry principle (such as the Principle of Indifference). Let 'event A' be 'the next shot bursts the balloon'. *Question*: What is the chance of the next shot bursting the balloon (token A)? The answer is of course: $P(A) = \pi r^2/a^2$.

Szabó (2007b, p.4) suggests that there is no particular reason to "look at how the physicist arrives at this result. What is important is that this equation does not, cannot, express a contingent fact of nature". The reason this cannot express a contingent fact of nature is because '$\pi r^2/a^2$' is operationally meaningful. It is an expression consisting of *ordinary*, *already-known* and *well-defined* physical quantities viz. the areas of the target and of the balloon, the ratio of which happens to be a quantity that satisfies Kolmogorov's axioms. However, '$P(A)$', the left hand side of '$P(A) = \pi r^2/a^2$', is not a known quantity. And, in so far as it is unknown what '$P(A)$' is, it is impossible to examine whether the equality is correct or not.[27] Thus, Szabó dissolves that the only possible interpretation of the chance assertion '$P(A) = \pi r^2/a^2$' is that $\pi r^2/a^2$ is the definition of $P(A)$. This entails that the precise form of $Ph(A) = \pi r^2/a^2$ *must be* $P(A) = \mu$ (area of the balloon/area of the target) = $\pi r^2/a^2$ where μ is the measurement of the corresponding *ordinary* and *well-defined* physical quantities. He concludes that what makes the chance assertion true is precisely the *ordinary* physical quantities of the area of the balloon over the area of the target, whose measurement yields the value '$\pi r^2/a^2$'.

Consider also how the physicalist 'no-chance' interpretation treats the typical example of a 'chancy' event; the toss of a fair coin.[28] The chance assertion is '$P(heads) = 0.5$'. This assertion is short for $P(heads) = \mu(X) = 0.5$, where μ is the measure of X. What makes '$P(heads) = 0.5$' true and thus meaningful is an *ordinary*, *already known and well-defined* physical quantity X. Of course, the physical system constituting the token under examination–the coin and its surrounding environment–is extremely complicated to say for certain which ordinary physical quantity this is. Yet the situation is familiar enough such that without knowing any specific details of the dynamics of the physical system we can entertain the hypothesis that X is the mass distribution of the coin. Thus, in the coin example we have completely different physical

[26] Roughly, an ergodic transformation map thoroughly 'blends' the elements of a given set. Consider for instance a bowl of water where a spoon of milk is dropped. Applying an apposite ergodic transformation map of the water will not allow the milk to remain in a local sub-region of the water. It will distribute the milk evenly throughout the water and at the same time the portion of the milk will not compress since milk preserves its density. See Dajani and Dirksin (2008) for an introductory discussion on ergodic theory.

[27] Measuring relative frequencies will not do because long run relative frequency is the interpretation of 'general probability' not of 'chance' and I consider the two as distinct concepts that are not definable in terms of one another. See discussion in Chapter 1.

[28] For how physics treats the toss of a coin see (Keller, 1986) and (Diaconis, 1998).

quantities from the balloon example but still ordinary ones, serving as the truth-makers of $P(\text{heads}) = 0.5$.

Generalising from the 'balloon example', Szabó concludes that 'chance' is a *collective* and *context-dependent* concept since, in the case of a completely different scenario, 'chance' refers to completely different but always *ordinary* physical quantities characterising the states of affairs of the particular physical system under examination. More simply, chance always refers to ordinary physical magnitudes or quantities.

In his 2007 paper the only ontological commitments Szabó makes for his interpretation is to a non-mathematical indispensability argument:

> We ought to have ontological commitment to all and only the entities that are indispensable to our best scientific theories. Mutatis mutandis, we ought to have ontological commitment to all and only the features of reality that are indispensable to our best scientific theories (Szabó, 2007b, p.4).

In his 2010 paper however, in a brief passage, Szabó notes about his interpretation:

> [W]hat we observe [...] is nothing but a kind of Humean supervenience [...] [chances] supervene on the collection of the particular facts of the actual history of the world [...] [T]his is true, no matter if the world is deterministic or indeterministic; either in the sense that the different time slices of the actual history are not functionally related; either in the sense that there exist other possible histories of the world besides the actual one; either in the sense of a more sophisticated branching structure of possible spacetime-histories [...] The truth or falsity of all meaningful statements about objective probabilities supervene on the Humean mosaic, where 'meaningful' is meant in a verificationist sense; that is, a statement is meaningful if it is expressible in terms of the Humean mosaic (Szabó, 2010, p.5).

In effect, the physicalist 'no-chance' interpretation of chance assertions says that chance *always* and in any given context corresponds to *ordinary* physical quantities. This is what turns chance assertions such as '$P(A) = \pi r^2/a^2$' and '$P(\text{heads})= 0.5$' into true propositions and thus makes them *operationally* meaningful. Importantly, the existence and the value of the ordinary physical quantities that correspond to chance in each particular context is independent of whether the laws of nature governing the gun firing and the path of the bullets, the toss of the coin and its trajectory, the spin of the roulette and its rotation etc. are deterministic (or not). Thus, 'chance' is a collective and a context-dependent term corresponding to different *ordinary*, physical quantities in any given context. The context is to be understood as the state of affairs constituting a specific space-time locus of the history of the world. The following analogy is, I believe, helpful to understand Szabó's interpretation: A chance assertions is like a snapshot of a specific place we derive by the operation of a camera. The snapshot depicts physical quantities present at the capturing moment when the button of the camera is pressed.

5.3.1 Szabó's 'no-chance' & Chance Assertions in QM and SM

Let me now turn to the applicability of Szabó's account to the chance assertions of our paradigmatic probabilistic physical theories of quantum and statistical mechanics respectively. The Physicalist 'no-chance' interpretation is not applicable to chance assertions in *standard* quantum mechanics (Szabó, 2007b). The reason is simple and has nothing to do with determinism (or not). Consider the quantum probabilistic assertion (the Born rule): Consider that a given quantity O has an associated operator $\widehat{O}$ written as $\widehat{O} = \sum_i o_i \widehat{\Pi}(i)$ where o_i are the distinct values of the operator and $\widehat{\Pi}(i)$ projects into the subspace of states with eigenvalue o_i. In case that the quantity O is measured on a quantum system with state $|\psi\rangle$, then: (i) the only possible outcomes of that measurement are the eingenvalues o_i of the operator and, (ii) the probability that the measurement results to o_i is $\mathrm{P}(O = o_i) = \langle\psi|\widehat{P}(i)|\psi\rangle$. The right hand side of this chance assertion is not a well-defined physical quantity or, at least it is not well-understood in the way the 'coin toss' or the 'inflated balloon' cases are. More precisely, as long as it remains a mystery what the physical meaning of 'wave function' is, we cannot consider it as the definition of the left hand side $\mathrm{P}(O = o_i)$ because in this case we don't know what '$\mathrm{P}(O = o_i)$' stands for, nor we understand what the physical meaning of '$\langle\psi|\widehat{P}(i)|\psi\rangle$' is.[29]

Let me discuss two ways the physicalist 'no chance' interpretation can be applicable to chance assertions in statistical mechanics. First, consider the probability postulate of Boltzmann-type statistical mechanics: 'P (state is in S) = S/Γ', saying that the 'chance' that the state of a system being in sub-region S of Γ equals S/Γ. In terms of the interpretation by Szabó (2007a) this would be short for: P (state is in S) = $\mu(S)/\mu(\Gamma) = S/\Gamma$ where S and Γ are regions of state space associated with the physical system under investigation and μ is a measure of the 'volumes' of the regions, usually a Lebesgue measure. The claim that there is a correspondence between the volumes according to the measure in the state-space and the quantities themselves is typically justified on the basis of symmetries. Alternatively, we can follow the conventionalist approach to physical theories and take the assumption of symmetries to function as the principle of coordination that guarantees the correspondence between the quantities themselves and the volumes measured in the state-space. Given the correspondence is conventionally established, we can consider that these ordinary physical quantities e.g. temperature, pressure etc. that the volumes measured in the state-space correspond to are the truth makers of the assertion '$P(S) = S/\Gamma$'.[30]

Second, we can apply the 'physicalist no-chance' interpretation to chance of Szabó (2007b) to the probabilistic assertions in statistical mechanics in terms of the method of arbitrary functions. The idea of using the mathematical method of arbitrary functions to understand the probabilities of statistical mechanics is to find a basis for the probability of an outcome in the properties of the physical dynamics that produce it. This would allow to derive probability distributions

[29]'Quantum probabilities' are discussed in the next chapter.

[30]See Chandler (1987) for an introduction to Statistical Mechanics and Ter Haar (1955) for a general discussion regarding its foundations.

out of physical dynamics without appealing to probabilistic facts (Myrvold (2016b), Strevens (2009), Strevens (2011), Werndl (2016)). Roughly, the mathematical idea is that the dynamics of certain systems are such that a wide range of initial probability distributions can be taken and the systems' evolution will result into distributions that assign approximately the same probabilities to some statements about that system. The reason is that small uncertainties about initial conditions evolve to greater uncertainties about the macroscopic variables of their system at a later time such that, as the system evolves, the initial conditions become negligible or they are 'forgotten' (Myrvold, 2016b, p.28). In brief, the dynamics of certain systems are such that different initial probability distributions will result to approximately the same probabilities about some statements of that system as the system evolves at a later stage.

Certain systems where statistical mechanics successfully apply to exhibit such behaviour. Myrvold (2016b) gives the example of an isolated system such as a bowl of hot water with an ice cube in it which, at that state, is out of thermal equilibrium. Suppose that it is left alone until thermal equilbrium is reached. Then, its former state is buried so deeply in the details of its microstate such that no feasible measurement can yield substantial information about its former state. For systems that have this feature, a wide range of initial probability descriptions p evolve through Liouville's equations into distributions that result in probabilities that are indistinguishable from those given by the equilibrium distribution.

Another example of a physical system that exhibits such a behaviour is provided by Strevens (2011): consider a wheel of fortune with an equal number of equally sized sections painted black and white. The wheel is given a certain velocity and when it comes to rest, a fix pointer reveals the outcome, either black or white. If one is asked what is the probability of the outcome 'black' and what is the probability of the outcome 'white', the intuitive answer is that the probability of each is one half. Strevens (2011) suggests that a closer examination of the system can justify these answers since they reveal that the system can be treated in terms of the method of arbitrary functions. First, regarding the dynamics of the wheel, that is, how initial velocities result to certain outcomes, Strevens (2011) notes that they have the feature or property he calls 'microconstancy'. That is, given small ranges of initial velocities, the proportion of these velocities that lead to the outcomes 'black' and 'white' is approximately one half for each. Second, one needs to take into account the manner in which the wheel is prepared in a certain initial velocity; such preparation can be modelled by a probability distribution p over initial velocities. The problem with the modelling of this preparation is that one's knowledge about the initial probability distribution is very limited i.e. the way the wheel is spun may correspond to very different initial probability distributions. The problem of limited information about the initial probability distribution can be neglected, or more precisely, this lack of information will not matter if it is *assumed* that all possible probability densities p one may employ have the property that Strevens (2011) calls 'macroperiodicity'. That is, they do not change severely on a very small region. When the dynamics of the system is microconstant and the probability density p is macroperiodic, the probabilities assigned to outcomes 'black' and 'white' will be approximately one half respectively. This will be the case even if the initial probability distributions are very different, given the assumption that all possible probability densities one could use are macroperiodic. If that's the case, then the probability of outcomes 'black' or 'white' is determined by two properties of a physical system of the wheel of fortune: the microconstancy of its physical dynamics in conjunction with the macroperiodicity of its physical probability distribution over initial conditions.

Of course, much depends on the assumption of the macroperiodicity of the probability density. One possible way to accommodate such an assumption is to consider it as a principle of coordination whose acceptance is a prerequisite that enables the mathematical manipulation of the physical system under examination in terms of the method of arbitrary function. In other words, the acceptance that probability densities are macroperiodic is a prerequisite for treating the wheel of fortune in terms of the method of arbitrary functions. For the current purpose, the important philosophical question is *how* to interpret the probabilities of the method of arbitrary functions. As Myrvold (2016b, p.29) puts it, this method does not generate probabilities out of nothing. Rather, the main idea is that a probability distribution over initial conditions is transformed into a probability distribution over conditions at a later time, via the dynamical evolution of the system. Thus, to use this method to understand the probabilities in statistical mechanics one first needs to answer the question that concerns the status of the initial probability distributions; that is, to interpret the initial probability distributions.

The physicalist interpretation by Szabó (2007a) allows for an objective or ontic interpretation of the initial probability distributions. First, given our very limited knowledge regarding the initial probability distribution we can follow Poincaré (2003) and consider that the choice of one probability distribution over another is ultimately based on a convention of our own. Second, given we consider the macroperiodicity of the probability density as a principle of coordination, the convention in choosing the initial probability distribution over another won't matter since it won't impact the probabilities assigned to each outcome. Then, following the physicalist interpretation by Szabó (2007a) we can interpret the initial probability distributions as corresponding to ordinary physical quantities characterising the particular situation at hand. Thus, the concept of probability is considered reducible to non-probabilistic ordinary physical quantities such that their precise meaning will depend on the context of application. In a nutshell, interpreting the probabilities of the method of arbitrary functions in following Szabó's interpretation, then these probabilities are nothing more than ordinary physical quantities characterising particular physical situations under examination. Werndl (2016, p.26) points out that this reading is fully objective or ontic since these ordinary physical quantities are part of the causal nexus of the world, they support counterfactuals about future predictions etc. To conclude, as far as the method of arbitrary functions can be used in physical systems that statistical mechanics successfully apply to, we can interpret their probabilities as reducible to non-probabilistic ordinary physical quantities depending on the physical situation at hand.

Also, the physicalist 'no-chance' interpretation of the chance assertions in statistical mechanics has similarities with a remark from Poincare in his discussion on chances in statistical mechanics:

> (I)n the kinetic theory of gases, we find the well-known laws of Mariotte and of Gay-Lussac, thanks to the hypothesis that the velocities of the gaseous molecules vary irregularly, that is to say, by chance [...] And, what is more extraordinary still, my answer will be right [...] Chance, then, must be *something more than the name we give to our ignorance* [...] it is clear that the information that the calculation of probabilities supplies will not cease to be true when the phenomena are better known (Poincaré, 2003, p.66) [My emphasis].

Poincare seems to support that there must be something objective about the probabilities in statistical mechanics since even if we knew all the properties of the particles constituting let's say a particular gas, we would have derived the same predictions for the thermodynamical system as those predicted by statistical mechanics. Paty (1990) suggests that a possible way to interpret Poincare is that he considers that the probabilities of statistical mechanics can be expressed in an objective manner by deeming that probability theory provides a theoretical way of selecting the physical quantities of the system required for its thermodynamic description.[31] This reading of Poincare's view on chance in statistical mechanics is objective in the sense he notes in the aforementioned extract; that is, the truth value of the probabilistic assertion won't change no matter if a finer description of the physical system were available. In other words, the physical quantities selected by *applying probability theory* are genuine quantities of the system no matter if the physical system is known up to an even finer description. Szabó makes a similar claim through the following example:

> You are waiting for the next train in a subway station. If you knew the exact timetable, you could make predictions like 'The next train will arrive in 3 minutes.' If you don't know the timetable but only know that the trains come in every 5 minutes, you can make less ambitious claims. For example, you can say that 'I will wait less than 5 minutes'; or you can predict the following result of a long-run experiment: 'Providing that the moments at which I enter to the station are uniformly distributed in time, the long-run average of my waiting time is 2.5 minutes.' Now, the validity of these claims does not change if you get know the timetable (Szabó, 2010, p.5).

One way to interpret both views is that in the cases they describe, the chance assertions refer to the world 'out there' and this is true no matter if a finer description of the situation is available. The assertion itself does not express lack of knowledge. For Poincaré, 'chance' *refers*

[31] In the terminology of Chapter 1, this a case of *applying* the mathematical theory of probability to non-mathematical quantities; in this case to physical quantities of the system that enables its thermodynamic description.

to 'something more than the name we give to our ignorance' whereas for Szabó 'chance' refers to ordinary, already known, and well-defined physical quantities. Either way, the feature that makes both accounts objective is the consideration that even if a finer description of the system becomes available, the truth value of the chance assertion remains unaltered. The truth makers of the chance assertions, the 'something more than the name we give to our ignorance' of Poincaré and 'the ordinary already-known physical quantities' of Szabó are all present in the run of the experiment.

To conclude, since a coherent analysis of 'propensity' has not been found (see Chapter 2), allow the use of the term 'propensity' for Szabó's ordinary physical quantities and for Poincaré's remark that chance is 'something more than the name we give to our ignorance'. Since these physical quantities/magnitudes are–certainly for Szabó–not modal, allow calling them 'Humean propensities'. These propensities, being ordinary magnitudes or quantities, supervene on the Humean mosaic.

5.4 Humean Propensities

This section introduces the Humean propensity analysis of the concept of chance. This analysis is nothing more than a complementary reading of a version of the BSA and Szabó's physicalist 'no-chance' accounts of chance. It suggests that the Humean propensity proposal captures many of the intuitions we usually assign to the concept of chance, especially in the physical sciences.

Let's begin by identifying some common features shared by the BSA and the 'no-chance' accounts: First, Szabó's Humean propensities can have non-trivial values irrespective of determinism (or not). So too, under certain modifications the BSA can deliver chance assertions with non-trivial values irrespective of determinism (or not). Second, Lewis' canonical BSA of chance holds as a metaphysical necessity and in 'no-chance' interpretations, chance assertions *do not/cannot* express a contingent fact about nature; this is the reason they can only be interpreted as *operational definitions*.[32] Third, both the BSA as well as the 'no-chance' interpretation do not consider chance as a *fundamental property* of an event. Rather, they hold that chance supervenes on properties of the Humean mosaic.

The suggested strategy for combining the BSA and the physicalist 'no-chance' accounts is the following: First, Humeanism about chance is assumed; that is, it is assumed that whatever that chance is, it supervenes. Second, the BSA is considered to be 'fixing the reference' of chance in our world without revealing the nature of the quantity or magnitude the concept refers to. In other words, it is considered that the *contingent* probability laws of our world merely 'fix the reference' of chance in our world. The following question is then asked: What could BSA chances refer to and what could they possibly mean?

[32]See footnote 4 for the claim that at least in its canonical conception the BSA holds as a metaphysical necessity.

The proposal is to approach this question using Bridgman's guide which says that "we must remain aware of the operationalist join in our conceptual framework"(Bridgman, 1927, p.24). Approaching the BSA chance assertions though this lense reveals two things: (i) they must correspond to Szabó's ordinary (non-modal), already known and well-defined physical magnitudes I dubbed 'Humean Propensities' and (ii) their *basic* meaning corresponds to *the set of operations* the particular Humean propensity–the one the probability laws of our world refer to–is measured by. In a nutshell, the Humean propensity account is nothing but a proposal to operationalise the concept of BSA chance. That is, to approach BSA chance assertions through 'Bridgman's guide' remarking that operationalist joints play an indispensable role in our conceptual structure and they have a constitutive role to play in the meaning of any given concept.

Let's now see how the analysis of BSA chance assertions in terms of Humean propensities scores with regard to the pre-theoretical intuitions usually assigned to the concept of chance. Consider interpreting BSA chance assertions such as 'the chance that the next shot bursts the balloon is $\pi r^2/a^2$' or 'the chance that the next toss of a coin lands heads is 0.5' in terms of Humean propensities.

Single-cases: Humean propensities are objective and meaningful in single cases; they are present in each sequential repetition of the situation e.g. each time a fair coin is tossed the experimental set up has its Humean propensity. In the particular case, the hypothesis is that the Humean propensity is the mass distribution of the coin. Each time a bullet is shot, the physical system has its Humean Propensity; e.g. in the balloon example, the Humean propensity corresponds to the area of the target and the balloon. Also, it is trivially true that Humean propensities supervene on the Mosaic and as such they are parts of the causal nexus of the world and thus support countefactuals about future predictions. These Humean propensities in some *intuitive sense* and in some cases and under certain conditions indicate the 'tendency' of the physical system as a whole to 'burst the balloon', to 'land the coin heads', to 'stop the roulette on black' etc. The main difference between Humean and hard propensities is that the Humean ones can be, at least in principle, defined through direct operational procedures. Most importantly, they are Humean as they do not require that "there are more things in heaven and earth that physics has dreamt of" (Lewis, 1994, p.474).

Best System Analysis: It is of no surprise that Humean propensities capture Lewis' *implicit* characterisation of BSA chance assertions when remarking that "I can see, dimly but well enough, how knowledge of frequencies and symmetries and best systems could constrain rational credence" (Lewis, 1994, p.484). Humean propensities are ordinary physical quantities and in some cases may reflect physical symmetries of the situation under examination e.g. the physical symmetry of the mass distribution of a *particular* fair coin. Under certain conditions–for Szabo these conditions can't be decided on a priory grounds but the BSA seems to capture such a relation by the virtue of 'fit'–the relative frequencies of the *event type* converge to the value of the Humean propensity; to the truth maker of 'Ch(heads) = 0.5'.

(In)determinism: Humean propensities are objective and meaningful in each run of the experiment irrespective of determinism (or not). Humean propensity is of the likes of areas of balloons over targets, mass distribution of coins, phase spaces of volumes etc. It is to things like these the probability laws of the best system(s) are referring to when uttering that '$Ch(A) = x$'. It is things like these that make '$Ch(A) = x$' true. And it is the *set of operations* of measuring things like these that provides the *basic* meaning of BSA chance assertions. In general, the existence and the value of the Humean propensity in any given context is not influenced by whether the dynamics of the situation under examination are deterministic (or not). That is, in contrast to hard propensities, the existence and the value of Humean propensities has nothing to do with whether the probabilities entail by the best systematisation of our world are dynamical or they are due to probability distributions over initial conditions.

'General probability': The important question for scientific practice is how chances are related to long-run frequencies, to general probabilities. In Chapter 1, I have argued that the concepts of chance and of general probability are not metaphysically related and as such any relation between the two, if there is one, is a contingent feature of the world and must be established on posterior grounds. Nevertheless, it is an empirical fact that Humean propensities have been found to relate to relative frequencies. Consider again the balloon example by Szabó (2007b). In such scenario, we can maintain for instance that the relative frequency of the *event type* 'the shot bursts the balloon' will be approximately equal to the value of the Humean propensity $\pi r^2/a^2$ if we assume that the size of the balloon remains constant in the sequential repetitions of the experiment. This is because we already know as a fact of kinematics that the shots are uniformly distributed i.e. by employing the apposite ergodic transformation map. Of course, by changing the size of the balloon during the sequential repetitions of the experiment the relative frequency of *event type* A will not converge to $\pi r^2/a^2$; that is to the Humean propensity. Nevertheless, if the size of the balloon remains constant then the truths of the chance assertion 'Ch(the next shot bursts the balloon) = μ (area of the balloon/area of the target) = $\pi r^2/a^2$ are in correspondence with the truths of the statement regarding the relative frequencies of the *event type* 'the shot bursts the balloon'. The correspondence however *is not* ensured by some metaphysical postulate. Rather, in the particular example the relation between the two is derived as a fact of kinematics and by the assumption that the size of the balloon remains constant in each sequential repetition of the experiment.

When a direct operational definition is not available–in practice this is usually the case–we could calculate the value of Humean propensity by measuring the relative frequencies of the event *type* that the *token* under consideration is thought to belong to and hope that 'nature is kind'; that is, that the value of the actual relative frequency of the event *type* is approximately equal to that of the Humean propensity. For instance, in the 'tossing of a fair coin' case, the sequential repetitions of the situation can be such that the value of Humean propensity X–the *hypothesis* is that X is the mass distribution of the coin–is approximately equal to the long-run relative frequency of *event type* 'heads'. Yet the fact that we considered the BSA to fix the 'reference of chance' in our world, and since the BSA is contacted to 'fit' the frequencies we could expect that, if 'nature is kind', the value of the Humean Propensity of the *token* 'heads' would be approximately equal to the long-run relative frequency of the *event type* 'heads'.

5.5 Conclusion

I have considered that the BSA 'fixes the reference' of the concept of chance in our world. I have then applied Bridgman's operationalist philosophy to develop 'Bridgman's guide' in order to understand the concept of chance within the BSA. I have concluded that the truth makers of BSA chance assertions must be the ordinary physical quantities in Szabo's physicalist no-chance interpretation I called Humean propensities. Since the Humean propensity interpretation of 'chance' is a hybrid of the BSA and of Szabó's physicalist 'no-chance' accounts, it enjoys many of the advantages of both.

The term 'Humean propensities' is being used as I consider the current proposal as a Humean version of modal propensity interpretations. The proposed account however brings propensities down to earth; that is, on the Humean mosaic. Still, they do what the hard propensity interpretations attempt to do; that is, they provide meaningful objective single-case probabilities. True, they are operationally meaningful but they are still meaningful. In general, if the modal propensities allow for an objectivist scientific knowledge as Miller (1995) claims then I cannot think of any reason that the Humean propensities will not. Humean propensities supervene on *ordinary* quantities or magnitudes and thus: (i) they describe how the world is, they are parts of the causal nexus of the world and as such they support counterfactuals regarding future prediction, (ii) no essential reference to epistemic notions like 'rationale credence' is required for making sense of the chance assertions in physical theories.

This is, I propose, the route to get a basic understanding of the concept of chance. Of course, one may find such an understanding unsatisfactory. Or even worse, to consider that as the expression goes, 'the baby is thrown out with the bathwater'. The baby seems fine to me; the Humean propensity analysis captures many of our pre-theoretical intuitions about chance, especially in the physical sciences. Admittedly, the baby won't necessarily have 'chance as a guide to rational life'. Nevertheless, the proposed account does not forbid such a guide to life either. It merely avoids making any essential reference to 'rational credence' in the analysis of chance.[33]

In the next chapter a case is made that chance may not be the relevant concept of objective probability for probabilistic assertions in standard quantum mechanics. Rather, 'quantum proba-

[33]The question regarding the relation between the proposed analysis and the PP or the NP; that is, under what conditions, if any, the Humean propensity analysis of chance becomes compatible with the PP or the NP, is an interesting question to be left for another time. Such a relation, if any, does not have essential impact on the Humean propensity analysis of chance.

bilities' are to be categorised as 'general probabilities'. To be more precise, the claim will be that Humeans have solid philosophical reasons to think so.

CHAPTER 6

'COMMITTED' HUMEANS AND 'QUANTUM PROBABILITIES'

> *"You know, the most amazing thing happened to me tonight [...] I saw a car with the license plate ARW 357. Can you imagine? Of all the millions of license plates in the state, what was the chance that I would see that particular one tonight? Amazing!"*
>
> — Attributed to Richard Feynman

It is often argued that quantum phenomena cannot be understood within the framework of the classical-relativistic (or pre-quantum) 'world view', with this causing trouble for the metaphysics of Humean Supervenience (Karakostas (2009), Maudlin (2007b), Hall (2016)). Suppose calling this claim the Common View (henceforth CV). One of the main arguments for CV derives from the results of the Bell/Aspect experiment. Roughly, the claim is that the 'quantum probabilities' in the Bell/Aspect experiment cannot be interpreted as long-run relative frequencies of *event types* in a Humean manner because such a reading violates Bell's inequalities. If that's the case, a *realist* interpretation of the quantum phenomena in the Bell/Aspect experiment is usually considered to entail a departure from the 'classical relativistic world view'. This chapter critically reflects upon this conclusion.

In rough terms, the overall argument of the chapter is the following: the results of Bell/Aspect experiments are considered as strong support in favour of the CV. Yet, Bell's theorem (inequalities)–that eventually led to Bell/Aspect results–makes a number of assumptions, one of which is that of 'Measurement independence'. Measurement independence is the assumption that there is no correlation between the postulated 'hidden variables' and the measurement settings in a quantum experiment; that is, the measurement settings of the quantum experiment are 'probabilistically independent' of the 'elements of reality' and the measurement outcomes of the experiment. 'Committed' Humeans have good reasons to suspect that measurement independence can be violated. In addition, the 'super-deterministic' *objective* reading of Bell/Aspect results can reproduce the observable frequencies of the experiment in a manner that is in perfect harmony with both the 'classical relativistic world view' and with the metaphysics of Humean Supervenience. Therefore, an objective interpretation of Bell/Aspect results does not require for 'committed' Humeans to deviate from or even alter their philosophical convictions.

Section 6.1 briefly describes the basic tenets of the classical relativistic world view and those of some of the post-quantum realist alternatives. *Section 6.2* notes the intended fit of the metaphysics of Humean supervenience with the classical relativistic 'world-view' and dubs what a 'committed' Humean stands for. *Section 6.3* describes the 'standard' formulation of the EPR argument. *Section 6.4* claims that the EPR argument is open to two interpretations depending on how one reads the 'free will' it assigns to experimentalists. *Section 6.5* describes Bell's theorem, its assumptions and the results of Bell/Aspect type of experiments that Bell's theorem has eventually led to. *Section 6.6* focuses on the assumption of measurement independence with which Bell essentially substitutes the assumption of 'free-will' in the canonical EPR argument. It discusses two arguments in favour of the assumption of measurement independence: (i) the argument for 'free will' and (2) the argument against 'cosmic conspiracy'. It then makes a case for the usually neglected from literature 'super-deterministic' reading of Bell/Aspect results that bites the 'cosmic conspiracy' bullet. *Section 6.7* describes the 'Kolmogorov's censorship hypothesis' of Szabó (1995) and the two distinct interpretations of the 'quantum probabilities' in Bell/Aspect results that this allows for: the *property interpretation* and the *minimal interpretation*. It is noted that the different 'no-go' theorems for Einstein-local HVTs presuppose the 'property interpretation' with this making a substantial difference since the 'minimal interpretation' allows one to interpret 'quantum probabilities' in a perfectly coherent manner–both conceptually and technically–as *objective* long-run frequencies of measurement outcomes relative to different measurement setups without deviating from the classical relativistic world view nor from the metaphysics of Humean Supervenience. Notwithstanding, the cosmic 'conspiracy' this reading implies.

6.1 The Classical World-View and 'Post-Quantum' Realist Alternatives

This section briefly describes the basic tenets of the classical relativistic world view and the 'post-quantum' realist alternatives.

The classical relativistic world view is based on three main principles: (1) *locality*, there is no direct causal connection between spatially separated events; no information travels faster than the speed of light, (2) *determinism*, every event is uniquely determined by the pre-history of its backward light cone where 'event' stands for a completely detailed description of the state of affairs in a given space-time region, of a definite piece of the history of the world, (3) *markovianity*, all past information is encrypted in the present state of affairs (Szabó (2007a), Hofer-Szabó (2017)).

On the other hand, the realist or objectivist 'post-quantum' alternatives are, in some sense, all peculiar. The *first* alternative is to commit to an irreducibly indeterministic universe where 'quantum probabilities' are to be understood either as irreducible propensities or as some sort of primitive chances that cannot be defined any further.[1] In rough terms, the idea is to postulate irreducible chances or genuine propensities as objective features of reality and consider those as by-products of a non-deterministic world and responsible for observed statistics in Bell/Aspect type of experiments.[2]

The *second* alternative is to maintain determinism but drop locality; that is, to allow for some sort of 'telepathic', faster than the speed of light, causal connection between spatially separated events; what Einstein called 'spooky' action at a distance. This is the main idea behind the non-local Bohmian mechanics, also called the De Broglie-Bohm Theory or Pilot Wave theory. Very briefly, according to Bohmian mechanics, elementary particles e.g. electrons, have at every time a definite position travelling in accordance with an equation of motion that includes a wave function that evolves in accordance with the Schrödinger equation (Tumulka, 2017). Bohmian mechanics predicts observable probabilities–essentially relative frequencies–identical to those predicted by standard quantum mechanics. One suggestion is that 'Bohmian probabilities' are to be interpreted as dispositions of possible corpuscle configurations manifesting themselves.[3] Because the theory is fundamentally non-local–the 'quantum potential' that locally governs a particle's behaviour depends on the simultaneous coordinates of other distant particles–has led many authors to consider that the theory is incompatible with the special theory of relativity (Seevinck (2010), Maudlin (2011), Norsen (2011)). For instance, Bell (2004, p.172) writes for "an apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory".[4]

The *third* alternative is to stipulate that there exist many parallel universes while considering standard quantum mechanics both fundamental and complete, maintaining both determinism and locality. This is the main idea behind the so-called Multiverse or Everettian interpretation of quantum mechanics. It supports an ontic reading of the 'wave function', denying that it ever 'collapses'. Reality splits into different branches every time quantum states diverge into different 'possibilities'. The superposition of states is seen as slices of a universal wave function diverging from each other as the wave function evolves in accordance with Schrödinger's equation without ever 'collapsing'. In this sense, all possible alternative histories are considered to represent real universes. That is, the theory holds that there is an extremely large number of universes existing

[1] For such views see Suárez (2007), and Ghirardi, Rimini and Weber's (GRW) objective collapse theory (Ghirardi, 2002) and for a discussion see Frigg (2009).

[2] As discussed in Chapter 3, Belnap and Green (1994) make a strong case that commitment to indeterminism requires commitment to ontic (or irreducible) modalities. Also, Chapter 3 has argued that indeterministic interpretations are threatened by circularity.

[3] For a discussion of the different interpretations of probability in Bohmian mechanics see Callender (2007), Maudlin (2007a) and Dürr and Ehmann (2017).

[4] For a detailed analysis of Bohmian Mechanics see Cushing (1994), Berndl et al. (1995), Goldstein (2012), Maudlin (2011), Tumulka (2017).

in the robust physical sense. According to this interpretation of quantum mechanics, 'quantum probability' is a measure of the 'bundles of trajectories' through the state space.[5]

I suppose it is not hard to see why Salmon (1998, p.279) in his discussion of the different interpretations of quantum mechanics concludes that "the situation is obviously desperate". This chapter won't examine these alternatives. Its aim is modest. Namely, it is to suggest that at least as far as the Bell/Aspect experiment is concerned, 'committed' Humeans have good reasons to resist CV and its implications regarding the interpretation of Bell/Aspect probabilities.

6.2 The Humean Project and the Classical Relativistic World View

This section describes the Humean Project and notes its intended fit with the classical-relativist world-view. In closing it states what a 'committed' Humean stands for.

As discussed in Chapter 4, Lewis has formulated the Humean project around the metaphysical doctrine of Humean Supervenience that: "all there is to the world is a vast mosaic of local matters of particular fact, just one little thing and then another" (Lewis, 1987, p.ix). Suppose interpreting the doctrine of Humean Supervenience from the perspective of 'Down to Earth Humeanism'; that is, a commitment to the doctrine of Humean Supervenience and to strict Actualism reading of modality *de re*.[6] The doctrine of Humean Supervenience is then divided into two theses. *Thesis 1:* All the truths about *the* world supervene on the distribution of perfectly natural properties and relations in *the* world. Everything supervenes on a *single* physical realm and can be expressed in physical terms.[7] *Thesis 2:* The perfectly natural properties and relations in the world are intrinsic properties of point-sized objects and spatio-temporal relations. At the foundations of physical reality what we have is a mosaic of local particular matters of fact and regularities of occurrences without any necessary connection between them (Lewis, 1987, p.ix). This thesis is to be read as a feature of the 'mind-independent' world. It states two things. *First*, that everything there is in the world *globally* supervenes on the microphysical domain.[8] The microphysical domain is the distribution of intrinsic properties of point-sized objects and spatiotemporal relations such that fixing the point-sized objects of the world 'automatically' fixes everything, all the properties, of that world. *Second*, it says that the fundamental space-time structure of the world is relativistic;

[5]For a comprehensive discussion see DeWitt (1970), Vaidman (2014), Wallace (2012), Wallace (2016).

[6]The argument of this chapter–as far as I can tell–holds for 'Possible Worlds Humeanism'; that is, for the commitments to the Humean Supervenience and to Modal Realism. I focus on Down to Earth Humeanism since this is the version of Humeanism that I endorse. For a discussion of 'Down to Earth Humeanism' and 'Possible Worlds Humeanism' see Chapter 4.

[7]The concept of supevenience is understood here as a relation between classes of physical properties. Take A and B be such classes. A supervenes on B when there is no difference in A-properties without some difference in B-properties; A-properties cannot be altered unless B-properties are altered (Lewis, 1986b).

[8]One way to define global supervenience is the following: macro-properties globally supervene on micro-properties if and only if for any worlds w_1 and w_2, if w_1 and w_2 have an identical worldwide pattern of distribution of micro-properties, then they also have an identical worldwide pattern of distribution of macro-properties (McLaughlin and Bennett, 2018).

that is, it is comprised of the four dimensions of space-time (Oppy, 2000). *Thesis 2* is intended to fit nicely with the general relativity theory, and especially with the claim that the four dimensions of space-time constitute the fundamental structure of physical reality. In general, the metaphysical doctrine of Humean Supervenience nicely complements the three pillars–locality, determinism and markovianity–of the classical relativistic world-view.

Consider a 'committed' Humean one who convicts to all of the aforementioned. A 'committed' Humean holds *thesis 1*: All the truths about the world supervene on the distribution of perfectly natural properties and relations in that world, and *thesis 2*: the perfectly natural properties and relations in the world are intrinsic properties of point-sized objects and spatiotemporal relations. Consequently, the aim of the Humean project for a 'committed' Humean is to express *everything* found in *the* world in a manner compatible with the metaphysics of Humean Supervenience. This can be carried out by providing truth conditions for all *contingent* truths in terms of the Humean mosaic.

The problem that quantum phenomena impose to 'committed' Humeans is the following: quantum phenomena like those studied in Bell/Aspect type of experiments are considered–for one reason or another–to provide experimental evidence against the classical-relativistic 'world view' and therefore to conflict especially with *thesis 2* of the canonical conception of the doctrine of Humean Supervenience.

6.3 The EPR Argument

This section describes the Einstein-Podolsky-Rosen thought experiment (henceforth EPR) that eventually has led to Bell/Aspect experiments and results. The EPR thought experiment aims to show that quantum mechanics cannot be a complete description of physical reality.[9] The standard formulation of the EPR argument goes roughly like this (*figure 1*):

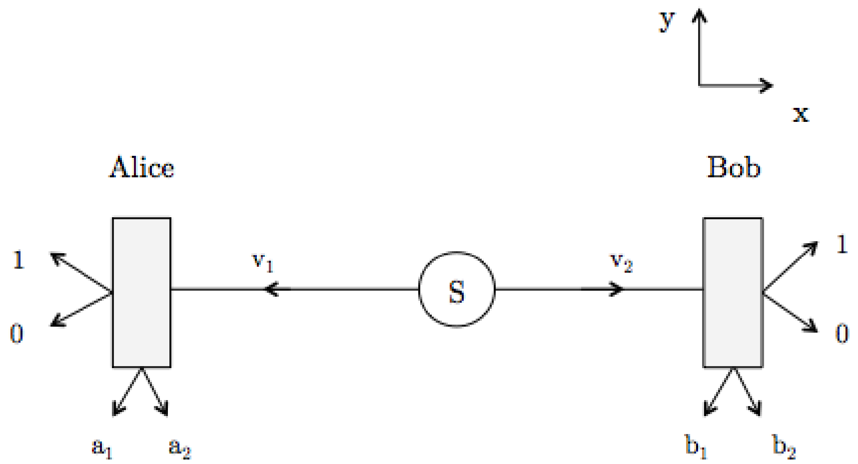

Figure 6.1: The EPR Thought Experiment

Assume a physical system [S], call it the source, consisting of two ‘entangled’ particles v_1 and v_2, initially interacting with each other.[10] Later on, they are separated and travel into opposite directions away from the source [S]. In each direction there is one observer; Alice on the left of the source and Bob on its right, such that v_1 travels towards Alice and v_2 towards Bob. There are two possible measurement settings for each; Alice can decide between measurement settings a_1 and a_2 and Bob between b_1 and b_2. For each measurement setting there are two possible measurement outcomes: 0 when no particle is detected, and 1 when it is. If Alice measures the spin of v_1 in the y-axis (measurement settings a_1) finding out that the spin is up, then due to ‘entanglement’ she knows in advance that if Bob measures the spin on the same basis

[9]As Fine (2017) points out, what is considered among researchers as the standard formulation of the EPR argument is due to Bohm and it differs significantly from the original formulation of Einstein-Podolsky-Rosen. This chapter follows the standard formulation since it is this version that is examined in Bell’s theorem.

[10]It is very subtle how to give a precise analysis of ‘entanglement’. We follow Schrödinger who states that ‘entanglement’ is the phenomenon “[Where] two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own [...] By the interaction the two representatives [the quantum states] have become entangled" (Schrödinger, 1935, p.555).

(y-axis); that is, if he chooses measurement setting b_1 he will find the spin being down. If it is further assumed that it is up to Alice and her 'free will' to measure the spin of the particle in any orientation she likes, then, according to EPR, it follows that at all times there must be an 'element of reality' that determines Bob's measurement outcome.

That is, given that Alice can predict Bob's measurement outcome with certainty and since no signal can travel faster than the speed of light (locality), it follows that there exists an 'element of reality' that determines Bob's measurement outcome. Since this information regarding this 'element of reality' does not enter into the quantum mechanical description of the system, the EPR concludes that quantum mechanics is an incomplete description of such a physical system. In other words, since the measurement of v_1 determines the outcome of v_2 even when the measurements are causally separated, and given than no signal can be transmitted faster than the speed of light then there *must be* an 'element of reality', a 'hidden variable' of some sort, that determines the measurement outcome of v_2. They write:

> If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity'. (Einstein et al., 1935, p.777).

This conclusion is known as EPR's Criterion of Reality and aims to show that the indeterminacy expressed by the probabilistic predictions of QM is epistemic and that quantum phenomena are 'deep down' Einstein-local deterministic.

In a nutshell, the EPR argument is based on the following premises. *Premise 1*: quantum mechanics predicts a strong correlation between distant measurements for certain quantum states ('quantum entanglement'). *Premise 2*: locality; no signal travels faster than the speed of light. *Premise 3*: the choice of the direction of the analyser is up to one's 'free will' and it can be decided right before the measurement takes place and while the particles are already flying. *Thus*, it must be the case that each particle has a yet unknown 'property' or 'element of reality' over and above the quantum mechanical description that determines its polarisation for *any* direction of analysis. Of course, the EPR *does not* imply that one can predict let's say the spin of v_2 in all directions at the same time due to the fact that one *cannot* measure the spin of v_1 in all directions at the same time.

6.4 The Experimentalists' 'Free Will'

This section focuses on two interpretations of the EPR argument, depending on how one understands the 'free will' the standard formulation of the argument assigns to the experimentalist. To be clear from the outset, Bell's theorem substitutes the assumption of 'free will' in the EPR argument with the assumption of measurement independence. Yet, 'free will' understood as the ability to do (e.g. measure) otherwise provides *sufficient but not necessary* conditions for measurement independence; that is, measurement independence can be justified by accepting this notion of free will, yet the latter is not a *necessary* condition for measurement independence to hold.

In relation to this, the aim of the following discussion on free will is simply to note that the concept of free will understood in the folk manner i.e. as the ability to do or measure otherwise, that provides sufficient yet not necessary conditions of measurement independence, is excluded by the initial formulation of the EPR argument since the argument presupposes determinism. The compatibilist reading of free will does not provide a justification of the violation of measurement independence but not sufficient conditions for it either. Also, some of the worries expressed against common cause explanations of the Bell/Aspect results this chapter aims for are formulated in terms of its incompatibility with the folk conception of free will. For instance, Ladyman (2000, p.854) says that " in order to prove Bell's theorem we have to consider counterfactual states of measurement apparatus [...] [T]here have to be facts about such modal matters as what would have been unaffected had some parameters been different, and we have to assume that we have a free choice in our experiments". In addition, what may be physically relevant from the distinction between the two conceptions of free will is that the folk conception requires some sort of modalities or counterfactual definiteness; that is, it requires that a hypothetical world where one did measure otherwise is a physically meaningful concept. On the other hand, the compatibilist concept of free will can be defined even if one accepts a strict actualist ontology. While the latter does not entail a violation of measurement independence, it could motivate one who shares such ontological commitments and aims in maintaining local determinism, the Common Cause Principle etc. to reconsider the assumption of measurement independence as a possible way out of Bell's theorem.

Hence, let me consider two arguments from the free will literature that illustrate this point. First, the *consequence argument* for the incompatibility between determinism and 'free will' understood as the ability to *do* otherwise. Second, *the compatibilist analysis of 'free will'* by Frankfurt (1988) where 'free will' is understood in terms of the ability to *desire* to do otherwise. The claim would be that when a deterministic universe is assumed like in the EPR case, the physicists performing Bell/Aspect type of experiments lack the 'freedom of the will' that provides *sufficient* conditions for measurement independence. Nevertheless, they can very well have Frankfurt's free will. The latter does not, in any way, entail by itself a violation of measurement independence but it does not forbid it either.

As discussed in the previous section, 'free will' appears in one of the premises of the 'standard' formulation of the EPR argument. Consider what 'free will' could possibly mean in such scenario. The EPR is constructed based on the classical relativistic world view whose one feature is determinism. That is, every event is *uniquely* determined by the pre-history of its backward light cone. Thus, the 'free-will' that the argument assigns to experimentalists must be compatible with determinism. If 'free will' is understood in the folk manner; that is, as the ability to measure otherwise, the EPR argument involves two mutually exclusive assumptions. *Assumption 1*: every event is *uniquely* determined by the pre-history of its backward light cone. *Assumption 2*: one has the ability to measure otherwise as if one's measurement was not *uniquely* determined by the pre-history of its backward light cone. If that's the case, EPR's conclusion that each particle has a 'hidden property' determining the polarisation for *any* direction of analysis does not follow. On the other hand, if the argument gives a compatibilist reading on 'free will' -this seems to be the most natural interpretation of the argument given the assumption of determinism- then the conclusion logically entailed by the premises is that there is an 'element of reality' over and above the quantum mechanical description of the physical system that determines the polarisation of the particle for *its only possible* direction of analysis. If determinism is true, there is only one possible direction of analysis and this is the actual one.[11]

The Consequence Argument: the definition of 'free will' is usually given in terms of the principle of alternative possibilities which states that: a person's act is free if and only if that person could have done (e.g. measure) otherwise; that is, free will is considered as the ability to do otherwise (O'Connor and Franklin, 2018).[12] The consequence argument by Van Inwagen (1983) notes the incompatibility between determinism and free will understood as the ability to measure otherwise. For the purpose of describing the consequence argument, consider the definition of determinism in terms of possible worlds semantics by Earman et al. (1986, p.13): Let W be the class of all physically possible worlds. The world $w \in W$ is deterministic if and only if for any world $w_i \in W$ it is the case that: if w and w_i are in the same state at some time t_0 then they are in the same state at all times t.

Consider the following scenario that describes the main idea of the consequence argument: Alice has the ability to choose or to do otherwise of x at time t if and only if it was possible that, given that everything is fixed up until t–the determinism thesis–Alice chooses or does otherwise at t. In possible worlds semantics, determinism is the thesis stating that all deterministic possible worlds sharing a common past and identical laws of nature will have identical futures. Assume that Alice is in a deterministic world W and goes for a vanilla ice cream at time t. By the definition of determinism it follows that in any possible world W' with the same past and laws of nature up to t with W, Alice must have the same future; that is, Alice going for a vanilla ice cream. Thus, Alice does not have the ability to do otherwise and consequently lacks the 'freedom' of doing otherwise. Van Inwagen (1983, p.16) writes:

> If determinism is true, then our acts are the consequences of the laws of nature and

[11] In fact, it is not clear whether all compatibilist accounts of 'free will' will do. Landsman (2017) argues that the notion of 'free will' described by the 'strong free will theorem' of Conway and Kochen (2009) is essentially Lewis' compatibilist notion of free will. He goes on and establishes a contradiction between the assumptions based on which Bell's theorem formulates the EPR scenario and the assumptions of the compatibilist account of free will á la Lewis.

[12] There are multiple ways to spell out such a modal claim but consider the categorical analysis that fits the Humean Project.

> events in the remote past. But it is not up to us what went on before we were born, and neither is it up to us what the laws of nature are. Therefore, the consequences of these things (including our present acts) are not up to us.

The consequence argument has the form of a conditional proof: it assumes determinism and given some additional principles it concludes the absence of ‘free will’. The argument involves three principles. *First*, the ‘no choice’ principle, stating that if one has no choice regarding x, and no choice whether if x, then y, one has no choice regarding y. This principle expresses the idea that if one has no control over certain things then one does not have control over the consequences of those things. The intuitions of the principle are clear: if y is an inevitable consequence of an x and x is out of one’s control, then y is out of the agent’s control. *Second*, one has no choice about events which happened in the distant past. *Third*, one has no choice about what the laws of nature are. Plugging the *second* and the *third* principles onto the *first*, the following conclusion is derived: No control over the laws of nature and one’s past entails no control over how one acts.

Suppose that x stands for the state of the universe right after the big bang and q stands for Alice’s choice regarding the measurement set-up in an Aspect-like experiment to be performed five minutes from now. By the No Choice principle, if Alice has no control (or choice) over the state of the universe right after the big bang and no control over the laws of nature, and given the laws of nature along with the state of the universe at a given time determine a unique future–the determinism thesis–it follows that Alice has no control over her ‘decision’ of the measurement set up in Bell/Aspect experiments. She doesn’t have control on the initial state of affairs right after the big bang nor the laws of nature at that time. Determinism in conjunction with the aforementioned principles determine a unique future.[13] In a nutshell, the argument claims that given the incompatibility of determinism with ontic possibilities, it follows that determinism is also incompatible with free will as the ability to do otherwise.

Frankfurt’s Compatibilism: On the other hand, ‘free will’ can be characterised as the ability to *desire* otherwise. This is the seminal idea of Frankfurt’s Compatibilism. His analysis of ‘free will’ is based on his theory of free action according to which: A’s ϕing is free if and only if A’s ϕing is P, for some property of actions P (Frankfurt (1969), Frankfurt (1988)). The compatibilist question is then reformulated as follows: if determinism is true, could any actions have property

[13] A common objection to the consequence argument is that it provides a very restrictive analysis of the ability to do otherwise. For instance, Lewis argues that the conclusion of the consequence argument that free will amounts to the ability to break a law of nature is open to two readings (Lewis, 1981). The *strong reading* says that Alice is able to do something such that, if done, it would cause a law of nature to be broken. The *weak reading* says that Alice is able to do something such that, if done, a law of nature would be broken. Lewis argues that compatibilists only require the weak thesis. In Lewis’ theory of counterfactuals, the antecedent ‘if done’ leads Alice to consider possible worlds where choosing different measurement settings from setting 1 is actually possible, while at the same time all things that could be kept the same as in the actual world are kept so (Lewis, 1979). Nevertheless, the conclusion ‘a law would be broken’ refers to the actual world. What is important is that the world where Alice chose something different than setting 1 is *not* the actual world (Beebee, 2013). Bell/Aspect experiments however are about the actual world. Even if the compatibilism of Lewis (1981) is accepted, it is questionable whether the free will it guarantees is compatible with the assumptions of Bell’s theorem (see Landsman (2017) for an argument that it is not).

P? If the answer were affirmative, then Frankfurt's theory would be providing an argument for the possibility of compatibilism between determinism and free will.

Roughly, his theory distinguishes between the following: *First-order desire:* a desire to perform some action e.g. Alice desires to choose measurement setting A in a Bell/Aspect type of experiment. *Will:* an effective first-order desire e.g. Alice's desire is to choose measurement setting A and she does so. *Second-order desire:* a desire to have a certain desire e.g. Alice desires that she should desire to choose measurement setting A. *Second-order volition:* a desire for a certain desire to be one's will e.g. Alice desires that her desire to choose measurement setting A is effective in bringing her to choose measurement setting A. Frankfurt proposes that 'free will' is analogous to 'free action'. Since freedom of action is analysed in terms of desires, freedom of will can be analysed in terms of desires too (Frankfurt, 1982, p.20). The freedom of action is the freedom to do what one wants to do. Analogously, the freedom of will is the freedom to will (desire) what one wants to will (desire). In Frankfurt's analysis of free will there is no need to require for free will to entail the ability to do otherwise, but only to have second order volition:

> It seems conceivable that it should be causally determined that a person is free to want what he wants to want. If this is conceivable, then it might be causally determined that a person enjoys a free will. There is no more than an innocuous appearance of paradox in the proposition that it is determined, ineluctably and by forces beyond their control, that certain people have free wills and that others do not (Frankfurt, 1988, p.25).

Consider the following Frankfurt-type example: At time t, Alice decides that she wants Bob to choose measurement setting A at some later time. At t_3 Alice hopes that Bob will choose measurement setting A on his own accord, otherwise she will force him to do so. At t_1, Bob is thinking whether to choose measurement setting A or B but after some thought he chooses measurement setting A. At time t_2, Bob chooses measurement setting A. For Bob, his decision seems completely free since the situation never reached t_3 and Alice did not have to force him to 'choose' measurement setting A. Yet, at t_1 it was already determined that Bob would be choosing measurement setting A. Bob must either decide to go for measurement setting A or not. Nonetheless, Bob's decision to choose measurement setting A appears to be free, just like our everyday decisions are. In effect, Bob has Frankfurt-type free will as long as he has the desire to measure in a certain way and his desire becomes effective; that is, Bob measures in the way he desires to measure.

It is therefore consistent to say that (at least some) experimentalists are determined to have Frankfurt-type free will. This notion of free will is compatible with a deterministic universe–every event is *uniquely* determined by the pre-history of its backward light cone where 'event' stands for a definite piece of the history of the world–and consequently with the EPR argument. Yet, it does not provide *sufficient* conditions for the assumption of measurement independence in Bell's version of the EPR.[14]

6.5 Bell's Theorem and Bell/Aspect's Results

This section discusses Bell's theorem, its assumptions and Bell/Aspect's type of experiments and their results. It debriefs the sense in which these results are considered to be providing experimental evidence against the classical relativistic world view.

Bell was considering whether it is possible to account for the EPR correlations in the framework of the classical relativistic 'world-view' (Fine, 1982). The famous inequalities of Bell have the form of a conditional proof: given some assumptions they prove that the outcome predicted by standard quantum mechanics is different from the outcome predicted by (Einstein-local) Hidden Variable Theories (henceforth HVT). Bell's version of the EPR makes three assumptions:

Assumption 1: There are 'hidden properties/ variables etc.' determining quantum properties e.g. spin. These 'hidden variables' are seen as describing *intrinsic* properties of the particles that *they carry while travelling from source to analyser*.

Assumption 2: Locality: "the direct causes (and effects) of events are nearby, and even the indirect causes (and effects) are no further away than permitted by the velocity of light" (Bell, 2004, p.239). In terms of the EPR scenario, Alice's measurement outcome only depends on her measurement settings and the 'hidden variable', while Bob's measurement outcome depends only on his measurement settings and the 'hidden variable'.

Assumption 3: There is no correlation between the postulated 'hidden variables' and the measurement settings in a quantum experiment. The measurement settings of the quantum experiment–including the 'choice' of the direction of analysis or the 'choice' of setting a random number generator in such and so manner–are *probabilistically independent* of the 'elements of reality' and the measurement outcomes (probabilistic independence is discussed shortly).

Bell has introduced an experiment whose outcome could be calculated by both standard QM and by EPR-type deterministic HVTs. Aspect performed the experiment and the results strongly confirmed the predictions of standard quantum mechanics against those predicted by EPR-type deterministic HTVs. All experiments that followed have violated Bell's inequalities and strongly confirmed the findings of Aspect.[15] Consider a simplified case of a Bell/Aspect type of experiment: As in the EPR case, there is apparatus constituted of the source in the middle, one detector

[14]As an illustration, consider Frankfurt's notion of free will as an analogy to the 'forced to be free' by Rousseau. One interpretation of 'forced to be free' is that in a democratic society, to be 'free' is to submit one's individual will to the general will where the latter is the will of the majority. For instance, one voted for X but the majority has voted for Y. After the election of Y, to be free is to submit your individual will for X to the general will for Y and take Y to be your will. In the free will case, if determinism is true, laws of nature and the initial conditions are more like a dictator. Individual will bears no impact on that of the dictators. The dictator determines the general will. However, if one happened to will what the dictator wills then she is free. She is determined to have a Frankfurt-type free will.

[15]For a comprehesive discussion on Bell's theorem and Bell/Aspect experiments see Fine (1982), Salmon (1998), Szabó (2010) and for a performed experiment where photons were 10.9 km apart see Tittel et al. (1998).

on its left and one on its right such that there are no known–mechanical, electromagnetic etc.–connections between the two detectors. Suppose that a random number generator is employed to set the switches on each detector either on 'measurement settings 1' (s_1), 'measurement settings 2' (s_2) or 'measurement settings 3' (s_3). In each run of the experiment the source is activated and 'entangled' particles are emitted. Unless something blocks the path between the source and the detector, *each* detector either flashes or it does not. That is, for each run of the experiment there are two possible measurement outcomes for each detector: measurement outcome 'flash' (o_f) and measurement outcome 'no flash' (o_n). Call '(anti)correlations' the following situations: when the detectors happen to be set to the same measurement settings e.g. [s_1 - s_1] or [s_2 - s_2] or [s_3 - s_3], then only one of the detectors flashes. Suppose calling the detector on the left 'Alice' and the one on the right 'Bob'; then for measurement settings [s_1 - s_1], [s_2 - s_2] and [s_3 - s_3] the measurement outcome is either $o_f A$ or $o_f B$.

Bell's theorem says that Einstein-local HTVs should predict anti-correlations approximately 2/3 of the times the experiment is run. According to Einstein-local HTVs, in each run of the experiment there are nine possible *measurement settings*:

$$[s_1 \text{ - } s_1], [s_1 \text{ - } s_2], [s_1 \text{ - } s_3], [s_2 \text{ - } s_1], [s_2 \text{ - } s_2], [s_2 \text{ - } s_3], [s_3\text{- } s_1], [s_3\text{- } s_2], [s_3 \text{ - } s_3]$$

And, for each run of the experiment there are eight possible 'pairs of states' of each particle (f stands for 'flash' and n for 'no flash'):

$$[f \text{ - } f \text{ - } f], [f \text{ - } f \text{ - } n], [f \text{ - } n \text{ - } n], [f \text{ - } n \text{ - } f], [n \text{ - } n \text{ - } n], [n \text{ - } n \text{ - } f], [n \text{ - } f \text{ - } f], [n \text{ - } f \text{ - } n]$$

For two out of the eight possible states–[f - f - f] and [n - n - n]–Einstein-local HVTs predict (anti)correlation for all nine measurement settings. For the remaining six possible pairs of states they predict (anti)correlation for five out of the nine possible measurement settings. For instance, for pairs of states [f - f - n] and [n - n - f], they predict (anti)correlation for measurement settings [s_1 - s_1], [s_1 - s_2], [s_2 - s_2], [s_2 - s_1] and [s_3 - s_3]. The same holds for the remaining four possible pairs of states. Thus, Einstein-local HTVs predict anti-correlation (approximately) 2/3 of the times the experiment is run: $(2/8 * 9/9) + (6/8 * 5/9) = 2/3$. However, when the experiment is run for many times one observes the following:

Observation 1: In total, half of the times the experiment is run the outcome is 'flash'; the relative frequency (N) of either $o_f A$ or $o_f B$; that is, $N(o_f A \cup o_f B)$, is (approximately) 0.5. And half of the times the experiment is run the outcome is 'no flash': the relative frequency of either $o_n A$ or $o_n B$; that is, $N(o_n A \cap o_n B)$), is (approximately) 0.5.

Observation 2: In, total, in half of the runs both detectors flash; $N(o_f A \cap o_f B)$ is (approximately) 0.5, and in half of the runs only one of them flashes; $N(o_f A \cup o_f B)$ is (approximately) 0.5.

Observation 3: In the runs where the detectors happen to be set to the same measurement settings–[s_1 - s_1], [s_2 - s_2], [s_3 - s_3]–only one of the detectors flashes; that is, the measurement outcome is either $o_f A$ or $o_f B$.

In general, as the standard quantum mechanics predicts, one observes anti-correlations approximately half of the times the experiment is run. Yet, Einstein-local HTVs predict anti-correlations approximately 2/3 of the times the experiment is run. A common interpretation of Bell/Aspect results is that they show the impossibility of Einstein-local HVTs. For instance, Dickson (2007, p.121) concludes that Bell's theorem essentially answers 'no' to the question whether "those correlations can be given a common cause explanation".

Yet, as mentioned, Bell's 'no go' theorem for common cause explanations of the observed (anti)correlations in Bell/Aspect results–and to that extend for Einstein-local deterministic HVTs–makes three assumptions. *Assumption 1*: there are 'hidden properties, variables, elements of reality etc. describing *intrinsic* properties of the particles that *they carry while travelling from source to analyser* determining quantum properties eg. spin. *Assumption 2*: locality; no information travels faster than the speed of light. *Assumption 3*: the measurement settings of quantum experiments–including the 'choice' of the direction of analysis or the 'choice' of setting a random number generator in such and so manner–are *probabilistically independent* of the 'elements of reality' and of the measurement outcomes of the experiment.[16] For the purpose at hand we only focus on *assumption 3*. This assumption comes under a variety of names: 'hidden autonomy' (Van Fraassen, 1982), 'no conspiracy' (Hofer-Szabó et al., 1999) (Placek and Wroński, 2009), 'free will' (Tumulka, 2007) 'measurement independence' (San Pedro, 2013). Consider using 'measurement independence' which is the most neutral in terms of connotation.

6.6 Measurement Independence

This section briefly describes the assumption of measurement independence noting that, at least for committed Humeans: (i) it is not as innocent as it may look, (ii) it affects how Bell's *Assumption 1* is to be interpreted e.g. whether (or not) the 'hidden variable' is an *intrinsic* property of the particles travelling from sources to analysers, from Alice to Bob etc. Reflecting on measurement independence, this section argues that it may alter what the Bell/Aspect results actually say about physical reality. The reason is that this assumption is essential for 'blocking' a common cause explanation of the Bell/Aspect results. It discusses two arguments in favour of the assumption of measurement independence: (1) the argument from 'free will' and (2) the argument against 'cosmic conspiracy'. It suggests that the 'super-deterministic' reading of Bell/Aspect results that bites the 'cosmic conspiracy' bullet should not be neglected, especially on purely philosophical grounds.

In rough terms, measurement independence is the assumption that fixing the 'hidden variable' does not restrict the choice of measurement settings and as such, the 'hidden variable' cannot influence measurement settings nor it is possible that there is a factor in their common past that correlates the two (Esfeld, 2015). In turn, this would allow one to examine *any* physical

[16] 'Probabilistic independence' is discussed shortly.

system without worrying that the examination is determined by what happened in the backward light cone of the experiment. More precisely, measurement independence holds *if and only if* the measurement settings of a quantum experiment–including the ‘choice’ of the direction of analysis, or the ‘choice’ of setting a random number generator in such and so manner, or that of a certain cosmological observation etc.–are *probabilistically independent* of the ‘elements of reality’ and the measurement outcomes of the experiment. Yet, the mathematical concept of (Kolmogorovian) ‘probabilistic independence’ does not come with an intuitive notion of ‘independence’ (Hajek and Hitchcock, 2016). As far as the mathematical concept of ‘probabilistically independence’ is the concern, two events, say A and B are probabilistically independent when the following relation holds: $P(A \cap B) = P(A)P(B)$.[17] A material (non-mathematical) conception of ‘probabilistic independence’ is a subtle matter. Kolmogorov (1933, p.9) writes on this: “we thus see, in the concept of independence, at least the germ of the peculiar type of problem in probability theory [...] one of the most important problems in the philosophy of the natural science is [...] to make precise the premises which would make it possible to regard any given real events as independent”.A material (non-mathematical) conception of probabilistic independence between real events e.g. sequential spins of a roulette, sequential repetitions of Bell/Aspect experiments etc., also requires a concept of causal independence.

Consider the following *necessary* and *sufficient* conditions for causal independence between real events provided by Reichenbach (1991): Let s_t and o'_t be real physical events occurring at times t and t' respectively, and t' is later than t. Then, s_t is a cause of o'_t *if and only if* $P(o'_t|s_t) > P(o'_t)$ *and* there is no event hv''_t that occurs at a time t'' earlier than or at the same time as t, such as hv''_t screens off o'_t from s_t. hv is said to screen off o from s if $P[s|(o \cap hv)] = p(s|hv)$. There is a spurious correlation when the relation $P(o|s) > p(o)$ holds even when s is not a cause of o; that is, the causal relation holds if both s and o are caused by a third factor hv. In effect, real events are causally independent if they are not the cause of each other nor there is a common cause in their common past e.g. a hidden variable, element of reality etc. connecting them.[18]

The assumption of measurement independence, as (Bell, 2001, p.111) observes, has metaphysical implications “disgrace indeed, to be caught in a metaphysical position! But it seems to me that in this matter I am just pursuing my profession of theoretical physics”. Yet, questioning the assumption of measurement independence in Bell’s theorem is usually considered as ‘conspiracious’ and maybe for good reasons. For once, it entails that our experiments are not freely ‘chosen’, but nature determines the sort of experiments we ‘choose’ to perform. Typically, two types of arguments are provided in favour of this assumption: (1) the argument that the experimentalists have the free will understood as the ability to do otherwise, (2) the argument against cosmic conspiracy. Both arguments appear to have the form of a *reductio ad absurdum*

[17]This relation can be generalised to any finite number of events. Let $C_1, C_2, ..., C_n$, be a finite number of events, these events are probabilistically independent when the probability of any intersection of the events factorises: for $1 < h \leq n$, and, $1 \leq g_1 < \cdots < g_k \leq n$: $P(C_g1 \cap \cdots \cap C_gh) = P(C_g1) \cdots P(C_gh)$.

[18]For a detailed analysis and a defence of Reichenbach’s Common Cause principle see Hofer-Szabó et al. (1999), Rédei (2002), Hofer-Szabó et al. (2013).

sort of argument; they accept the violation of measurement independence and show that it results to peculiar consequences.

For instance, an argument for free will in favour of measurement independence is that of Zeilinger (2010) who argues that if 'free will' is not granted to the experimentalists, the validity of science that always assumes the freedom of experimentalists would be undermined. And to this end, there would be no point asking questions regarding the workings of nature (e.g. by performing experiments), since nature would determine what our questions are. As it has been argued (section 6.4), the problem with this argument is that the EPR argument is incompatible with such a concept of free will to begin with; the EPR presupposes determinism. Also, there is a *reductio ad absurdum* argument that 'addresses' Zeilinger's worries: if the experimentalist do not have the folk concept of free will, then, at the end of the day, it is not up to them but up to nature to undermine the validity of science.

An argument for measurement independence in terms of the physical fact that its violation in Bell/Aspect experiments entails 'cosmic conspiracy'–or some sort of 'pre-established harmony'– comes from Maudlin (2014). Maudlin (2014, p.23) considers the physical implication that the violation of the assumption in Bell/Aspect experiments "would require a massive coincidence, on a scale that would undercut the whole scientific method" and argues that the assumption of measurement independence is something similar to Kant's notion of synthetic a priori; a necessary condition for the possibility of experience and for pursuing science in general. It is true that for measurement independence to be violated in Bell/Aspect experiments, this would require a massive coincidence. But it is not certain that it would undercut the whole scientific method. For instance, it could give one incentive to search for a theory that explains this massive coincidence.

Also, as Hofer-Szabó (2017, p.17) observers, the assumption of measurement independence is neither a priory nor a synthetic a priori truth. His characterisation of the assumption highlights this claim nicely: measurement independence is the assumption that experiments are the celebration of good business between two *independent* parties, humans and nature, both equally contributing to its success. Humans and nature are distinct and on equal footing, an agreement is made between the two to work together and to trust each other such that scientific knowledge is achieved. In support of his argument that measurement independence is neither a priory nor a synthetic priory truth, Hofer-Szabó (2017, pp.4-6) provides some classical scenarios where measurement independence is violated. One version of these scenarios is the following:

Consider an opaque piggy bank. The experimenter inserts two coins inside. Each coin is of different mass distribution and of different colour: one is gold and one is bronze. The experimenter sets up two measurement settings: (1) shake the piggy bank until a coin falls out and check its colour, (2) shake the piggy bank until a coin falls out, toss it and check whether it landed heads or tails. The measurement outcomes of the first setting are: the coin is gold and the coin is bronze. Those of the second setting are: the coin lands tails and the coin lands heads. Now, suppose that the coin of the second mass distribution is also 'slippery' and the experimenter 'chooses'–in Frankfurt's terms–to perform a colour measurement (setting 1) instead of a face one (setting 2) when the slippery coin happens to come out of the piggy bank. Suppose also that the characteristics of a coin being slippery and of having second mass distribution have a common cause; the coins with these two features have been made *in the same factory*. In such a scenario, measurement independence is violated. Measurement settings are not independent from the coin being slippery and having second mass distribution. There is a common cause that connects the two; that is, the factory that made the coins with these two characteristics. Of course, the piggy bank example does not show that the justification of measurement independence can be universally dismissed. What it does reveal is that whether or not the assumption holds is to be decided on a case by case basis; it is neither an a priori truth nor a synthetic a priori one.

If the assumption that measurement independence holds in Bell/Aspect experiments is dropped, the experimental findings can be understood in a coherent manner, consistent with the classical relativistic 'world view'. This reading of Bell/Aspect results is usually called 'super-determinism'. In effect, super-determinism denies that the measurement settings in Bell/Aspect experiments are entirely autonomous of the state of affairs in the backward light cone of the experiment. It implies a systematic correlation of all physical quantities in the visible universe, a cosmological scale 'conspiracy'. While this is indeed a peculiar implication, the remaining options do not fare much better: either determinism must be dropped, or it must be that there is action at a distance, or that there is an extremely large number of universes all existing in the robust physical sense, or that there is retro-causality. All of these peculiar options, super-determinism included, save the phenomena. It therefore seems that there is no way to tell which of them is true.

In addition, despite its peculiar implications, the basic philosophical idea of super-determinism is nothing new. It traces back to Parmenides' cryptic remark that the reality of the world is 'one being', and Spinoza's claims for 'the unity of all that exists' and 'the regularity of all that happens'. At least at the current stage, none of these options can be excluded on purely philosophical basis and as a result, the super-deterministic way out of Bell's theorem should not be so easily neglected. 'Conspiratory' or not, it allows to provide an objective reading of Bell/Aspect results maintaining both Einstein's causality–signalling into the past light cone is impossible–and the Common Cause principle of Reichenbach (1991, p.157): "If an improbable coincidence has occurred, there must exist a common cause".[19]

To conclude the discussion on the assumption of measurement independence, two types of arguments in favour of the assumption of measurement independence have been discussed. First, the argument that the experimentalists have the free will that allows them to measure otherwise. This would provide sufficient conditions for measurement independence. Yet, it has been argued (section 6.4) that such a notion of free will is excluded by the initial formulation of the EPR

[19]For a comprehensive discussion of a common cause explanation of the results of Bell/Aspect see (Brans, 1988), (Szabó, 1995), (Hofer-Szabó et al., 2013) and (Hess et al., 2016).

argument as it is incompatible with the assumption of determinism that the EPR presupposes. Second, the argument that highlights the physical fact that the violation of the assumption of measurement independence in a manner that superdeterminism requires, implies a cosmological scale 'conspiracy'; that is, that there is a persistent correlation of all physical quantities in the visible universe. This is true. It is also true that there is no a priori reason to think that there are no common cause mechanisms in the universe–plausibly tracing back to the Big Bang–that explain such a cosmic 'conspiracy'. The 'cosmic conspiracy' the super-deterministic reading entails is a bullet that one who aims to maintain determinism, local causality, the Common Cause Principle etc. in the construction of the theory of Bell/Aspect experiments may need to bite.

The core of this reading fits well with 'Down to Earth Humeanism', a conviction to the metaphysical doctrine of Humean Supervenience and to the strict Actualism reading on modality *de re*. An implication of these convictions is that there are no ontic possibilities in the universe and that alternative possibilities are epistemic. This is not to say that these convictions entail a violation of the assumption of measurement independence but certainly they do not forbid it either. The next section discusses how, in the case of a super-deterministic universe, quantum probabilistic assertions–at least as far as Bell/Aspect type of experiments are concerned–can be classified as 'general probabilities' instead of 'chances'.

6.7 'Quantum Probabilities'

This section describes the 'Kolmogorov censorship hypothesis' Szabó (1995) puts forward in his penetrating article, as well as the two distinct interpretations of 'quantum probabilities' in Bell/Aspect results that this allows.[20] These are the *property interpretation* of 'quantum probabilities' and the *minimal interpretation* of 'quantum probabilities'. Two main points are made. First, that the different 'no-go' theorems for Einstein-local HVTs presuppose the 'property interpretation'. Second, that the 'minimal interpretation' of 'quantum probabilities' allows one to interpret 'quantum probabilities' in a perfectly coherent manner, both conceptually and technically, as *objective* long-run frequencies of measurement outcomes relative to different measurement set ups.

In effect, the *property interpretation* remarks that the occurrence of a measurement outcome in each run of Bell/Aspect type of experiments reflects the existence of a certain property, element of reality etc. responsible for *that* measurement outcome. On the other hand, the minimal interpretation of 'quantum probabilities' says that there are no quantum mechanical properties, elements of reality, etc. corresponding to the outcomes of the measurements *one can* perform on quantum systems. It says that 'quantum probabilities' are nothing but classical (Kolmogorovian) conditional probabilities of measurement outcomes of quantum observables,

[20]For proofs of the Kolmogorov censorship hypothesis see Bana and Durt (1997), Szabó (2001a), and Redei (2010) for the proof that it holds not only for Hilbert space quantum mechanics, but also for general quantum probability theories based on the theory of von Neumann algebras.

where the conditioning events are the events of *choosing* to set up a measuring device to measure a *certain* observable.

The following 'standard' argument against Einstein-local HVTs is based on the property interpretation: If the EPR 'quantum probabilities' are understood as probabilities of *intrinsic* properties of the particles travelling from source to analyser, then the Clauser-Horne inequality that plays the same role as that of Bell is violated.[21] The Bell/Aspect experiments have shown that the EPR 'quantum probabilities' violate the Bell-Clauser-Horne inequalities. Consequently, EPR 'quantum probabilities' cannot be understood as relative frequencies.[22] Thus, there can not exist 'quantum' properties, elements of reality etc. whose relative frequency equals the 'quantum probability'. However, this conclusions *does not* follow if one adopts the *minimal interpretation* of quantum probabilities. Here is a simplified description of Szabó's argument for the distinction between the property and the minimal interpretation of 'quantum probabilities':

Suppose that quantum mechanics describes the following situation: Alice performs a measurement (m) in an entity on a state (s) and one possible outcome of that measurements is o. Alice repeats that experiment multiple times and counts how many times the outcome o occurs. Suppose that Bob manages to represent this situation in terms of Hilbert space quantum mechanics; he figures out a Hermitonian operator $\hat{m}$ corresponding to measurement m, a density operator $\hat{s}$ corresponding to state s and a measurement outcome o with a suitable projector $\hat{o}$ from the spectral composition of $\hat{m}$ such that the relative frequency of outcome (o) is '$tr(\hat{s}\hat{o})$'.[23] Suppose that Alice and Bob are debating how to interpret this '$tr(\hat{s}\hat{o})$'. Alice supports the *property interpretation* of quantum mechanical probabilities which says that '$P(O) = tr(\hat{s}\hat{o})$' where the left hand side of the equation denotes the probability that the entity the measurement is performed in has the *property* O. Upon reflection they realise that if the property interpretation is adopted then they have to abandon the relativistic classical world view for one of the more 'exotic' alternatives, some of which are briefly described in the beginning of the chapter (assuming they want an objective interpretation of the quantum probabilistic assertion). The property interpretation violates the Bell-Clauser-Horne inequalities: if the left hand side of the assertion '$P(O) = tr(\hat{s}\hat{o})$' is interpreted as P (the entity the measurement is performed in has the property O) then '$tr(\hat{s}\hat{o})$' cannot be interpreted as the relative frequency of outcome o because this reading violates Bell-Clauser-Horne inequalities. Alice is fine with the more 'exotic' alternative and she points out to Bob that absurd does not mean false. She is absolutely right.

On the other hand, Bob has every reason not to be convinced or simply to fail comprehending these alternatives suggesting instead the *minimal interpretation* of quantum probabilities which says that '$P(o|m) = tr(\hat{s}\hat{o})$', where the left hand side of the assertion denotes the conditional probability of the occurrence of outcome o given measurement m is performed. The minimal

[21]The conditional probabilities in Bell-Clauser-Horne inequalities are the corresponding absolute probabilities in the EPR. For the mathematical proof see Brans (1988) and Szabó (1995).

[22]For the proof see Pitowsky (1989, pp.27-31) and Szabó (2007a, pp.8-20).

[23]See Mackey (1957) for a discussion of Hilbert space quantum mechanics.

interpretation does not violate Bell-Clauser-Horne inequalities because it *does not* assume the existence of a property, of an element of relatity etc. that guarantees the measurement outcome o. That is, Bob, contra-Alice, *does not* assume that there are properties or elements of reality corresponding to the outcomes *one can* perform on that system. To be more precise, Bob's claim is that quantum mechanics does not describe such properties. This however implies that Bob *does not* need to explain the correlation between spatially separated occurrences of events observed in Bell/Aspect type of experiments in terms of such hidden variable, property, element of reality etc. since he denies that such property exists or, at least, he denies that quantum mechanics describes such hidden variable, property, element of reality etc. In contrast, for Bob–for the minimal interpretation–what there exist are real physical events corresponding to Alice performing measurements (s) and real events corresponding to measurement outcomes (o) where 'event' is understood in the sense of the theory of relativity; that is, as a definite space-time locus, as a totally detailed state of affairs in a given space-time region of the history of the world.

These two interpretations of 'quantum probabilities' and the viability of the minimal interpretation is the core of the 'Kolmogorov censorship hypothesis' of Szabó (1995). In effect, the Kolmogorov censorship hypothesis allows one to maintain that 'quantum probabilities' are just classical (Kolmogorovian) conditional probabilities of measurement outcomes of quantum observables, where the conditioning events are the events of *choosing* to set up a measuring device to measure a *certain observable*. This is a consistent, both technically and conceptually, interpretation of 'quantum probabilities' as conditional probabilities in a classical (Kolmogorovian) probability measure space without violating the Bell-Clauser-Horne inequalities.

Also, the analysis of the nature of an experiment by Hofer-Szabó (2017, pp.2-4) makes a convincing case that, especially for those of us who are sympathetic to the empiricist doctrine, the rationale of the minimal interpretation of 'quantum probabilities'–that the *event types* of measurement settings and measurement outcomes are indispensable parts of the analysis of the experiment–is not exclusive to quantum mechanical experiments; they are indispensable parts of the analysis of scientific experiments in general.[24]

Here is a brief description of his analysis of the nature of an experiment: In an experiment, one sets up the measurement apparatus in a certain way, performs the experiment, records the outcome of the measurement and repeats the procedure for a sufficiently large number of times. Every experiment has two indispensable *event type* categories: *measurement settings* and *measurement outcomes*. The instances of these *types* are the *tokens* collected from the repetitions of the experiment. These *event types* are the 'observables' of the experiment. Everything else imposed by the theory has to relate one way or another to the *types* of *measurement settings* and *measurement outcomes*. In order to make sense of the outcomes of the experiment, additional

[24]The empiricist doctrine holds that observation is the only window to the physical world; experience is the only access and thus the only source of information that we have regarding the external reality. Reason provides the means for making sense of the relations among the ideas acquired through experience. The truths about the 'world out there' that these ideas might reflect must ultimately boil down to sense experience (Markie, 2017).

event types are usually introduced. Call these sorts of *types* 'elements of reality'. The idea is that there are 'elements of reality' out there in the world leading to the change in the state of affairs from measurement settings to measurement outcomes. These 'elements of reality' *types* are the 'unobservables'; the experimenter has no direct access to them. The event algebra of the theory of the experiment is the Boolean combination of the measurement settings, measurement outcomes and 'elements of reality' *types* where each element of the algebra corresponds to a particular repetition of the experiment. Probabilities are calculated by counting the number of the repetitions of the experiment that instantiate certain elements of the algebra.

The role of empiricism in Hofer-Szabó (2017) analysis of an experiment is not to justify nor to suggest a violation of measurement independence. Rather, it points out that measurement settings and measurement outcomes are *indispensable* event types of the theory of an experiment since they are the only types one has direct access to. Now, if the assumption of measurement independence holds, one can neglect these measurement types and talk directly about the non-directly observable elements of reality types. That is, given measurement independence, the probabilistic relations between measurement settings and measurement outcomes will reflect probabilistic relations between the elements of reality. If the assumption does not hold, one cannot make such an inference.

In the case of the Bell/Aspect experiment, if measurement independence does not hold, then the assumption that there is an element of reality understood as an *intrinsic* property of the particles travelling from sources to analysers responsible for the measurement outcomes (Assumption 1) cannot be inferred either. Also, measurement independence is in conflict with other principles such as local determinism and Common Cause Principle. Suppose that one, for whatever reason, wants to maintain these principles in the construction of the theory of the experiment and decides to drop measurement independence. An analysis of the experiment with the following two features is available: (1) measurement settings and measurement outcomes are indispensable event types of the theory of the experiment and (2) quantum mechanics is incomplete, as it does not describe the element of reality responsible for the measurement outcomes. Rather, the element of reality is somewhere in the backward light cone of the experiment–plausibly tracing back to the Big Bang–serving as the common cause of measurement settings and measurement outcomes and being beyond the quantum mechanical description of the system.

Consider analysing the EPR scenario in the aforementioned manner; that is, by considering the *event types* of measurement settings and measurement outcomes as an indispensable part of the analysis of the EPR scenario as well as by considering that quantum mechanics *does not* describe the postulated 'element of reality'. The 'element of reality' *type* is somewhere in the backward light-cone of each run of the experiment with quantum mechanics being incomplete in the sense that it does not describe it:

There are two experimentalists, Alice and Bob, in analogous experimental situations. For simplicity the analysis will be based on Alice. The measurement settings *event types* of Alice are divided into the following two sub-categories: measurement setting 1 (s_1) when the particle is 'chosen' to be analysed along the x-axis, and measurement setting 2 (s_2) when the particle is 'chosen' to be analysed along the y-axis. In each repetition of the experiment the *token events* either belong to one or to the other *event type*. That is, each time the experiment is performed, Alice 'chooses' to measure either along the y-axis or along the x-axis such that the *token event* either belongs to s_1 or to s_2 *type*. Alice has two possible measurement outcomes for each measurement setting: o_1 when a particle is detected and o_2 when it is not. These are also *event types* and in each repetition of the experiment the *token event* either belongs to o_1 or to o_2 *type*. The experiment is run for many times and the *token events* of each run of the experiment are placed in their corresponding *event types*. From these *event types* the algebra of the experiment is built. The correlations between measurement settings and measurement outcome *event types* of the experiment using probabilities are expressed as follows:

(1) $P(o_1|s_1) = \frac{o_1 \cap s_1}{s_1} = N(o_1)$

(2) $P(o_1|s_2) = \frac{o_1 \cap s_2}{s_2} = N(o_1)$

(3) $P(o_2|s_1) = \frac{o_2 \cap s_1}{s_1} = N(o_2)$

(4) $P(o_2|s_2) = \frac{o_2 \cap s_2}{s_2} = N(o_2)$

The semantics that connects the theory of the experiment to physical reality goes like this: 'probability' is read as the long-run frequency. The probability sample space is all the *event types* of the experiment; *viz.* s_1, s_2, o_1, o_2. Each element of the algebra of the experiment corresponds to a *token event*, to a single run of the experiment. Each *token event* is read in the relativity theory sense; that is, as a completely detailed description of the state of affairs in a given space-time region, as a definite piece of the history of the world. After repeating the experiment for many times one categorises the *tokens* in terms of what one deems as the appropriate *event types* these *tokens* belong to. We interpret assertion (1) to say that the probability of detecting a particle (o_1) given analysing along the y-axis (s_1) is the number of the *token events* that belong both to o_1 and s_1 *event types* divided by the number of the *token events* that belong only to s_1 *event type*. We read (2) to (4) in the same manner. This analysis is for Alice. Likewise for Bob. This reading reproduces the results of Bell/Aspect in a manner compatible with the classical-relativistic 'world view' and without violating Bell-Clauser-Horne inequalities.[25]

Nevertheless, the minimal interpretation of quantum probabilities comes with the cosmic 'conspiracy' catch; that is, it implies a link–due to the operation of some common cause mechanisms plausible tracing back to the Big Bang–connecting all physical magnitudes at all energy scales in the observable universe. Recall that in order to avoid the Bell-Clauser-Horne inequalities, the minimal interpretation reconstructs quantum probabilities as classical Kolmogorovian conditional probabilities and *explicitly* ties the meaning of quantum probabilities to measurement settings' event types. Thus, the probabilities that relate to-–in all other respects—identical

[25]See Szabó (1995, pp.12-13) for a 'toy-model' illustrating that an Einstein-local HVT of Bell/Aspect type of experiments is not excluded by quantum mechanics.

instantiations of a particular experiment will have a different supervenience base depending on the way the measurement setting of the experiment is fixed. That is, the supervenience base of the probabilities resulting from the repetitions of the-—in all other respects—identical experiment would be different if the measurement settings of the experiment are fixed by using the experimentalists' 'free will' as the randomised mechanism, different if the measurement settings are fixed by using the decay of an unstable atom as a randomised mechanism, different if the measurement settings are fixed by using some cosmological observation in the early universe as our randomised mechanism, and so on. The explanation that the minimal interpretation provides for this peculiar consequence is that of super-determinism; that is, it arises because there are some interesting common cause mechanics in the universe systematically correlating *all* physical magnitudes of all scales in the observable universe, including the 'choices' involved in picking the randomised mechanism that fixes the measurement settings in quantum experiments. While this implication does not rule out the minimal interpretation-—there is no a priori reason to think that there are no common cause mechanics in the universe that explain such a cosmic 'conspiracy'—it is indeed an unsettling consequence that an advocate of the minimal interpretation of quantum probabilities has to accept. As discussed in Sec. 6.2, all interpretations of quantum probabilities have peculiar consequences, and the minimal interpretation is certainly not an exception.

To sum up the discussion on quantum probabilities, in the case that super-determinism holds in our universe, the minimal interpretation of 'quantum probabilities' allows one to consider that quantum probabilistic assertions are about *real/physical* relative frequencies of measurement outcomes, where the conditioning events are *real/physical* events of 'choosing' a measurement setting to measure a certain observable. In this manner, the EPR correlations and the Bell/Aspect results can be coherently–both conceptually and technically–interpreted as *objective* long-run relative frequencies of measurement outcomes relative to different measurement settings.

Notwithstanding, the minimal interpretation of quantum probabilities implies cosmic 'conspiracy'. If one is willing to accept that, she can interpret 'quantum probabilities' as *objective* general probabilities (aka long-run frequencies) and as such, to supervene on the mosaic in the Humean manner. Thus, at least as far as Bell/Aspect results are concerned, 'committed' Humeans can provide an objective interpretation of quantum phenomena as supervening on particulars, on tokens of the Humean mosaic that instantiates the *event types* of measurement settings and measurement outcomes. In terms of the distinction between two objective concepts of probability—general probabilities and chances-—quantum probabilistic assertions are to be categorised as general probabilities instead of chances.

6.8 Conclusion

This chapter has reflected on the 'common view' that quantum phenomena cannot be understood within the framework of the classical-relativistic (or pre-quantum) 'world view', with this causing trouble for the metaphysics of Humean Supervenience. The EPR thought experiment has been described, arguing that one gets two significantly different conclusions from the standard formulation of the EPR argument depending on how 'free will' is read. If it is read as the ability to measure otherwise then the argument involves the two conflicting metaphysical assumptions of determinism and of free will as the ability to measure otherwise. The conclusion of the EPR that there is a 'hidden element of reality' over and above the quantum mechanical description that determines the polarisation of the particle for *any* direction of analysis does not follow. The second interpretation of the EPR gives a compatibilist reading to 'free will' and leads to a significantly different conclusion. That is, there is a 'hidden element of reality' over and above the quantum mechanical description that determines the polarisation of the particle for *its*, instead of *any*, direction of analysis.

It is noted that the assumption of measurement independence in Bell's theorem that substitutes that of 'free will' in the canonical EPR argument has metaphysical implications. Sufficient conditions for measurement independence require metaphysical conviction to irreducible modalities; irreducible modalities could, potentially, grant the experimentalists the ability to measure otherwise and in consequence to provide sufficient conditions for measurement independence. Nevertheless, it has been argued that in the classical 'world-view' examined by Bell's theorem, there are no *sufficient* conditions for measurement independence. Without this assumption, nothing excludes the possibility of a super-deterministic reading of Bell/Aspect results. 'Conspiratory' or not, this reading indicates that the Bell/Aspect 'quantum probabilities' can be coherently–both technically and conceptually–interpreted in an *objective* manner as long-run relative frequencies relative to different measurement settings in the standard Humean manner. This of course has an unpleasant consequence: experimentalists performing Bell/Aspect experiments lack the 'free will' understood as the ability to do otherwise. On top of that, they may be parts of a cosmic 'conspiracy'.

Either way, what is important for our purposes is that 'committed' Humeans can give an objective interpretation of Bell/Aspect probabilities without deviating from or altering their philosophical convictions.

CHAPTER 7

CONCLUSIONS

The *central question* this study has investigated is the following: 'What could *probabilistic assertions* in physical theories possibly mean given one's commitment to their objectivity? Answering this question is important for at least two reasons. First, it is unclear how one can take the teachings of our currently best scientific theories seriously if they don't know what their *probabilistic assertions* mean. Second, one has to consent to a subjectivist reading of their probabilistic assertions which collides with the *desideratum* of an objectivist theory of scientific knowledge.

This study elaborated on how it would approach the *philosophical* task of providing an 'interpretation of probability'. Following the 'formalist' tradition, it considered the task of 'interpreting probability' as providing an analysis of the *probabilistic assertions* one derives by *applying* the mathematical theory of probability–for this study that of Kolmogorov–to physical (non-mathematical) phenomena. These probabilistic assertions were categorised into three classes, with each class corresponding to a distinct concept of probability: (i) 'subjective probability', (ii) 'objective general probability' and (iii) objective 'single-case probability' or 'chance'. It has been argued that each of these concepts requires a distinct interpretation, a distinct analysis. It has further suggested that probabilistic assertions in physical theories e.g. statistical and quantum mechanics, require an objective interpretation–either that of 'general probability' or of 'single-case probability'–given that an agreement is reached on which objective concept is suitable in each and every case.

It has been proposed that long run frequency provides the most tenable interpretation of 'general probability'. With the metaphysical doctrine of Humean Supervenience at its disposal, this study considered the concept of general probability as supervening on *token* events of the Humean mosaic instantiating certain event *types*. Moreover, it has been argued that none of the single-case propensity interpretations–hard and hybrid–provide a coherent analysis of the concept of single-case probability. Furthermore, it was suggested that an interpretation of 'single-case probability' that directly associates the concept with that of indeterminism–especially with the claim that quantum mechanics indicates that the world is indeterministic–is threatened with circularity.

Considering that the analysis of the concept of single-case probability depends on the concept of possibility, I have distinguished between two Humean reductions of modality *de re*–Down to Earth Humeanism and Possible Worlds Humeanism–that the Humean Propensity interpretation of the concept of 'single-case probability' this study argues for is relied upon. This interpretation is nothing more than a complementary reading of the Best System Analysis of 'chance' and Szabó's 'No chance' interpretation. The interpretation I proposed is called 'Humean Propensity' as it captures the features of the hard propensity interpretations whilst avoiding their problems, and without committing to the deterministic (or not) nature of physical reality. In brief, the Humean Propensity interpretation remarks that 'single-case probability' or 'chance' is defined as in a certain version of the BSA. After raising concerns with the tenability of the 'Subjectivist' and 'No-interpretation' guides, 'Bridgman's guide' to BSA *chance assertions* was proposed. As it has been proposed, 'Bridgman's guide reveals that the truth-makers of BSA *chance assertions* are the ordinary, already known, and well defined physical quantities of Szabó this study calls 'Humean propensities'. As a consequence of following 'Bridgman's guide', it is suggested that the basic meaning of BSA chance can be defined operationally. I claim that this interpretation captures most of the intuitions that one assigns to the concept of chance, especially in the physical sciences. Admittedly, this analysis does not justify the claim that the concept of chance is a 'guide to rational life'. Yet, it does not forbid it either. It just admits that it has no justification for it.

Lastly, a case was made that (especially) Humeans have solid philosophical reasons to reconsider whether 'chance' is the relevant concept of objective probability for the *probabilistic assertions* in standard quantum mechanics. That is, there are good reasons for one to consider that quantum *probabilistic assertions* are to be categorised as 'general probabilities'. The final chapter critically reflected on the common view that quantum phenomena cannot be understood within the framework of the classical-relativistic world view, with this causing issues to the metaphysics of Humean Supervenience. It was noted that the assumption of measurement independence in Bell's theorem has metaphysical implications. Sufficient conditions for measurement independence require metaphysical conviction to irreducible (ontic) modalities. Nevertheless, in the classical relativistic world-view examined by Bell's theorem, there can be no *sufficient* conditions for this assumption. Without this assumption nothing excludes the possibility of the super-deterministic reading of Bell/Aspect results. 'Conspiratory' or not, this reading shows that the Bell/Aspect 'quantum probabilities' can be coherently–both technically and conceptually–interpreted in a fully *objective* manner as long run frequencies, relative to different measurement settings in the standard Humean manner. Notwithstanding, the unpleasant consequence of the 'cosmic conspiracy' that the super-deterministic reading of Bell/Aspect implies is a bullet that an advocate of this view may have to bite.

Looking forward, future research prospects can build upon this study and the foundations it provided. I would like to explore the implications that the the Humean Propensity interpretation of the concept of single-case probability may have on the debate between ontological reductionism and pluralism. That is, to investigate whether it allows for an objective reading of the probabilistic assertions in special sciences, irrespective of determinism (or not). Also, I would be interested in examining the relation between the proposed Humean Propensity analysis of chance and 'rational' credence; that is, under what conditions, if any, can the Humean propensity analysis of chance serve as a 'guide to rational life'? Moreover, another ramification of the distinction between 'chance' and 'general probability' that I find to be an interesting potential research project is the observation that there is some sort of 'choice' to be made in terms of which is the suitable concept the *probabilistic assertions* of our currently best scientific theories are to be categorised as. This may have implications regarding what these theories with their probabilistic assertions tell us about the world; they may state one thing when their probability assertions are 'single-case probabilities' and another when they are 'general probabilities'. Also, I would be keen to further examine the advantages (or not) that different objectivist (realist) interpretations of 'quantum probabilities' bear on super-deterministic reading, or whether this is perhaps neglected in literature because of its unsettling implications.

All in all, I hope that this study has effectively signified that Humeans are in fact in place to *coherently* interpret both concepts of objective probability–the 'general probability' and the 'single-case probability' often called 'chance'. In a sense, this study was an attempt to demystify the concept of probability, to bring it back to earth, to the Humean mosaic. Whether satisfactory or not, this is up to the reader to decide.

Bibliography

David Z Albert.
Time and chance, 2001.

Leslie E Ballentine.
Propensity, probability, and quantum theory.
Foundations of physics, 46(8):973–1005, 2016.

Gergely Bana and Thomas Durt.
Proof of kolmogorovian censorship.
Foundations of Physics, 27(10):1355–1373, 1997.

Michael Beaney.
Carnap's conception of explication: from frege to husserl?'.
Carnap Brought Home: The View from Jena, Chicago, IL: Open Court, pages 117–50, 2004.

Michael Beaney.
Analysis.
In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, summer 2018 edition, 2018.

Helen Beebee.
Free will: An introduction.
Springer, 2013.

Claus Beisbart.
Good just isn't good enough: Humean chances and boltzmannian statistical physics.
In *New directions in the philosophy of science*, pages 511–529. Springer, 2014.

John S Bell.
Free variables and local causality.
In *John S Bell On The Foundations Of Quantum Mechanics*, pages 84–87. World Scientific, 2001.

John Stewart Bell.
Speakable and unspeakable in quantum mechanics: Collected papers on quantum philosophy.

Cambridge university press, 2004.

Nuel Belnap.
Branching space-time.
Synthese, 92(3):385–434, 1992.

Nuel Belnap and Mitchell Green.
Indeterminism and the thin red line.
Philosophical perspectives, 8:365–388, 1994.

Karin Berndl, Martin Daumer, Detlef Dürr, Sheldon Goldstein, and Nino Zanghì.
A survey of bohmian mechanics.
Il Nuovo Cimento B (1971-1996), 110(5-6):737–750, 1995.

Alexander Bird.
The dispositionalist conception of laws.
Foundations of Science, 10(4):353–370, 2005.

Seamus Bradley.
Are objective chances compatible with determinism?
Philosophy Compass, 12(8):e12430, 2017.

Carl H Brans.
Bell's theorem does not eliminate fully causal hidden variables.
International Journal of Theoretical Physics, 27(2):219–226, 1988.

Percy Williams Bridgman.
The logic of modern physics, volume 3.
Macmillan New York, 1927.

Mario Bunge.
Chasing reality: Strife over realism.
University of Toronto Press, 2015.

Craig Callender.
The emergence and interpretation of probability in bohmian mechanics.
Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics, 38(2):351–370, 2007.

Rudolf Carnap.
Logical foundations of probability.
1962.

David Chandler.
Introduction to modern statistical mechanics.
Introduction to Modern Statistical Mechanics, by David Chandler, pp. 288. Foreword by David Chandler. Oxford University Press, Sep 1987. ISBN-10: 0195042778. ISBN-13: 9780195042771, page 288, 1987.

Hasok Chang.
Operationalism.
Stanford encyclopedia of philosophy, 2010.

John Conway and Simon Kochen.
The strong free will theorem.
Notices of the AMS, 56(2):226–232, 2009.

Mikaël Cozic and Brian Hill.
Representation theorems and the semantics of decision-theoretic concepts.
Journal of Economic Methodology, 22(3):292–311, 2015.

James Crutchfield and Karoline Wiesner.
Simplicity and complexity.
Physics world, 23(02):36, 2010.

Erik Curiel.
Measure, topology and probabilistic reasoning in cosmology.
arXiv preprint arXiv:1509.01878, 2015.

James T Cushing.
Quantum mechanics: historical contingency and the Copenhagen hegemony.
University of Chicago Press, 1994.

Karma Dajani and Sjoerd Dirksin.
A simple introduction to ergodic theory.
2008.

Radin Dardashti, Luke Glynn, Karim Thebault, and Mathias Frisch.
Unsharp humean chances in statistical physics: A reply to beisbart.
In *New directions in the philosophy of science*, pages 531–542. Springer, 2014.

Bruno De Finetti.
Probability, induction and statistics: The art of guessing.
Wiley New York, 1972.

Bryce S DeWitt.

Quantum mechanics and reality.
Physics today, 23(9):30–35, 1970.

Persi Diaconis.
A place for philosophy? the rise of modeling in statistical science.
Quarterly of Applied Mathematics, 56(4):797–805, 1998.

Michael Dickson.
Non-relativistic quantum mechanics, January 2007.
URL `http://philsci-archive.pitt.edu/3321/`.
This paper is a preprint versison of a paper by the same title that has appeared in the Handbook of The Philosophy of Science, Philosophy of Physics, edited by Jeremy Butterfield and John Earman.

Dennis Dieks.
Reichenbach and the conventionality of distant simultaneity in perspective.
In *The present situation in the philosophy of science*, pages 315–333. Springer, 2010.

John Divers.
Possible worlds.
Routledge, 2006.

Patrick Dürr and Alexander Ehmann.
Probabilities in debroglie-bohm theory: Towards a stochastic alternative, January 2017.
URL `http://philsci-archive.pitt.edu/12859/`.

John Earman.
The "past hypothesis": Not even false.
Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics, 37(3):399–430, 2006.

John Earman and Wesley C Salmon.
The confirmation of scientific hypotheses.
Introduction to the philosophy of science, pages 42–103, 1992.

John Earman et al.
A primer on determinism, volume 37.
Springer Science & Business Media, 1986.

Albert Einstein, Boris Podolsky, and Nathan Rosen.
Can quantum-mechanical description of physical reality be considered complete?
Physical review, 47(10):777, 1935.

Adam Elga.
Infinitesimal chances and the laws of nature.
Australasian Journal of Philosophy, 82(1):67–76, 2004.

Edward Elliott.
Probabilism, representation theorems, and whether deliberation crowds out prediction.
Erkenntnis, 82(2):379–399, 2017.

E Allen Emerson and Joseph Y Halpern.
"sometimes" and "not never" revisited: on branching versus linear time temporal logic.
Journal of the ACM (JACM), 33(1):151–178, 1986.

Lina Eriksson and Alan Hájek.
What are degrees of belief?
Studia Logica, 86(2):183–213, 2007.

Michael Esfeld.
Reductionism today.
In *European Philosophy of Science–Philosophy of Science in Europe and the Viennese Heritage*, pages 89–101. Springer, 2014.

Michael Esfeld.
Bell's theorem and the issue of determinism and indeterminism.
Foundations of Physics, 45(5):471–482, 2015.

Gareth Evans.
Can there be vague objects.

Luke Fenton-Glynn.
Deterministic chance.
The British Journal for the Philosophy of Science, 61(1):51–80, 2009.

Luke Fenton-Glynn.
D. H. MELLOR The Matter of Chance.
The British Journal for the Philosophy of Science, 62(4):899–906, 07 2011.
ISSN 0007-0882.
doi: 10.1093/bjps/axr026.
URL `https://doi.org/10.1093/bjps/axr026`.

Luke Fenton-Glynn.
Unsharp best system chances.
2014.

Luke Fenton-Glynn.
Imprecise chance and the best system analysis.
Philosophers' Imprint, 19(23):1–44, 2019.

Arthur Fine.
Some local models for correlation experiments.
Synthese, 50(2):279–294, 1982.

Arthur Fine.
The einstein-podolsky-rosen argument in quantum theory.
In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2017 edition, 2017.

Harry Frankfurt.
The importance of what we care about.
Synthese, 53(2):257–272, 1982.

Harry G Frankfurt.
Alternate possibilities and moral responsibility.
The journal of philosophy, 66(23):829–839, 1969.

Harry G Frankfurt.
Freedom of the will and the concept of a person.
In *What is a person?*, pages 127–144. Springer, 1988.

James Franklin.
Resurrecting logical probability.
Erkenntnis, 55(2):277–305, 2001.

Michael Friedman.
Dynamics of reason.
2004.

Roman Frigg.
Grw theory (ghirardi, rimini, weber model of quantum mechanics).
In *Compendium of quantum physics*, pages 266–270. Springer, 2009.

Roman Frigg and Carl Hoefer.
Determinism and chance from a humean perspective.
In *The present situation in the philosophy of science*, pages 351–371. Springer, 2010.

Haim Gaifman and Marc Snir.
Probabilities over rich languages, testing and randomness.
The journal of symbolic logic, 47(3):495–548, 1982.

Maria Carla Galavotti.
Operationism, probability and quantum mechanics.
In *Topics in the Foundation of Statistics*, pages 99–118. Springer, 1997.

Giancarlo Ghirardi.
Collapse theories.
2002.

Ronald Giere.
Review of mellor's the matter of chance.
Ratio, 15(1):149–155, 1973a.

Ronald N Giere.
Objective single-case probabilities and the foundations of statistics.
In *Studies in Logic and the Foundations of Mathematics*, volume 74, pages 467–483. Elsevier, 1973b.

Donald Gillies.
Varieties of propensity.
The British journal for the philosophy of science, 51(4):807–835, 2000.

Donald Gillies.
Philosophical theories of probability.
Routledge, 2012.

Donald A Gillies.
Operationalism.
Synthese, 25(1):1–24, 1972.

Sheldon Goldstein.
Typicality and notions of probability in physics.
In *Probability in physics*, pages 59–71. Springer, 2012.

Sean Gryb.
New difficulties for the past hypothesis.
arXiv preprint arXiv:2006.01576, 2020.

Zalán Gyenis and Miklós Rédei.
Defusing bertrand's paradox.
The British Journal for the Philosophy of Science, 66(2):349–373, 2014.

Ian Hacking.
The emergence of probability: A philosophical study of early ideas about probability, induction and statistical inference.

Cambridge University Press, 2006.

Joshua Haddock.
The principal principle and theories of chance: Another bug?
Philosophy of Science, 78(5):854–863, 2011.

Alan Hájek.
“mises redux”—redux: Fifteen arguments against finite frequentism.
In *Probability, Dynamics and Causality*, pages 69–87. Springer, 1997.

Alan Hájek.
The reference class problem is your problem too.
Synthese, 156(3):563–585, 2007.

Alan Hájek.
Fifteen arguments against hypothetical frequentism.
Erkenntnis, 70(2):211–235, 2009.

Alan Hajek and Christopher Hitchcock.
Probability for everyone—even philosophers.
In *The Oxford Handbook of Probability and Philosophy*. 2016.

Alan Hájek et al.
Interpretations of probability.
2011.

Ned Hall.
Correcting the guide to objective chance.
Mind, 103(412):505–517, 1994.

Ned Hall.
Two mistakes about credence and chance.
Australasian Journal of Philosophy, 82(1):93–111, 2004.

Ned Hall.
David lewis’s metaphysics.
In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2016 edition, 2016.

Katherine Hawley.
Indeterminism and indeterminacy.
Analysis, 58(2):101–106, 1998.

Karl Hess, Hans De Raedt, and Kristel Michielsen.
Counterfactual definiteness and bell's inequality.
arXiv preprint arXiv:1605.04889, 2016.

David Hilbert.
Mathematical problems.
Bulletin of the American Mathematical Society, 8(10):437–479, 1902.

Christopher Hitchcock.
Probability and chance: Philosophical aspects.
2001.

Carl Hoefer.
The third way on objective probability: A sceptic's guide to objective chance.
Mind, 116(463):549–596, 2007.

Carl Hoefer.
Causal Determinism.
In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, spring 2016 edition, 2016.

Gábor Hofer-Szabó.
How human and nature shake hands: The role of no-conspiracy in physical theories.
Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics, 57:89–97, 2017.

Gábor Hofer-Szabó, Miklós Rédei, and László E Szabo.
On reichenbach's common cause principle and reichenbach's notion of common cause.
The British Journal for the Philosophy of Science, 50(3):377–399, 1999.

Gábor Hofer-Szabó, Miklós Rédei, and László E Szabó.
The principle of the common cause.
Cambridge University Press, 2013.

Dipankar Home and Matthew AB Whitaker.
Ensemble interpretations of quantum mechanics. a modern perspective.
Physics Reports, 210(4):223–317, 1992.

Leon Horsten.
Philosophy of mathematics.
In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, spring 2019 edition, 2019.

Colin Howson and Peter Urbach.
Scientific reasoning: the Bayesian approach.
Open Court Publishing, 2006.

Paul Humphreys.
Why propensities cannot be probabilities.
The philosophical review, 94(4):557–570, 1985.

Paul Humphreys et al.
Probability theory and its models.
In *Probability and Statistics: Essays in Honor of David A. Freedman*, pages 1–11. Institute of Mathematical Statistics, 2008.

Jenann Ismael.
A modest proposal about chance.
The Journal of Philosophy, 108(8):416–442, 2011.

Jenann Ismael.
How to be humean, 2015.

Allen Janis.
Conventionality of simultaneity.
In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2018 edition, 2018.

Vassilios Karakostas.
Humean supervenience in the light of contemporary science.
Metaphysica, 10(1):1–26, 2009.

Joseph B Keller.
The probability of heads.
The American Mathematical Monthly, 93(3):191–197, 1986.

John Maynard Keynes.
A treatise on probability.
Courier Corporation, 1921.

Jaegwon Kim.
Concepts of supervenience.
In *Supervenience*, pages 37–62. Routledge, 2017.

William Kneale.
Modality de dicto and de re.

In *Studies in Logic and the Foundations of Mathematics*, volume 44, pages 622–633. Elsevier, 1966.

AN Kolmogorov.
Foundations of the Theory of Probability: Second English Edition.
Courier Dover Publications, 1933.

Andrei N Kolmogorov.
On tables of random numbers.
Theoretical Computer Science, 207(2):387–395, 1998.

James Ladyman.
What's really wrong with constructive empiricism? van fraassen and the metaphysics of modality.
The British Journal for the Philosophy of Science, 51(4):837–856, 2000.

James Ladyman, James Lambert, and Karoline Wiesner.
What is a complex system?
European Journal for Philosophy of Science, 3(1):33–67, 2013.

Klaas Landsman.
On the notion of free will in the free will theorem.
Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics, 57:98–103, 2017.

Pierre-Simon Laplace.
Pierre-Simon Laplace Philosophical Essay on Probabilities: Translated from the fifth French edition of 1825 With Notes by the Translator, volume 13.
Springer Science and Business Media, 1825.

AJ Leggett.
The quantum measurement problem.
science, 307(5711):871–872, 2005.

David Lewis.
Anselm and actuality.
Nous, pages 175–188, 1970.

David Lewis.
Radical interpretation.
Synthese, pages 331–344, 1974.

David Lewis.

Counterfactual dependence and time's arrow.
Noûs, pages 455–476, 1979.

David Lewis.
A subjectivist's guide to objective chance.
In *Ifs*, pages 267–297. Springer, 1980.

David Lewis.
Causal decision theory.
Australasian Journal of Philosophy, 59(1):5–30, 1981.

David Lewis.
New work for a theory of universals.
Australasian journal of philosophy, 61(4):343–377, 1983.

David Lewis.
1980. a subjectivist's guide to objective chance. in his.
Philosophical Papers, 2:83–132, 1986a.

David Lewis.
On the plurality of worlds.
Oxford, 14:43, 1986b.

David Lewis.
Humean supervenience debugged.
Mind, 103(412):473–491, 1994.

David Kellogg Lewis.
Philosophical papers: Volume il.
1987.

Barry Loewer.
Determinism and chance.
Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics, 32(4):609–620, 2001.

Barry Loewer.
David lewis's humean theory of objective chance.
Philosophy of Science, 71(5):1115–1125, 2004.

Barry Loewer.
Counterfactuals and the second law.
2007.

Barry Loewer.
Why is there anything except physics?
Synthese, 170(2):217–233, 2009.

Edward Jonathan Lowe.
Vague identity and quantum indeterminacy.
Analysis, 54(2):110–114, 1994.

EJ Lowe.
Vague identity and quantum indeterminacy: further reflections.
Analysis, 59(264):328–330, 1999.

Ernst Mach.
Principles of the theory of heat: Historically and critically elucidated, volume 17.
Springer Science & Business Media, 1900.

George W Mackey.
Quantum mechanics and hilbert space.
The American Mathematical Monthly, 64(8P2):45–57, 1957.

Patrick Maher.
Betting on theories.
Cambridge University Press, 1993.

Peter Markie.
Rationalism vs. empiricism.
In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2017 edition, 2017.

Per Martin-Löf.
The definition of random sequences.
Information and control, 9(6):602–619, 1966.

Tim Maudlin.
What could be objective about probabilities?
Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics, 38(2):275–291, 2007a.

Tim Maudlin.
Quantum non-locality and relativity: Metaphysical intimations of modern physics.
John Wiley & Sons, 2011.

Tim Maudlin.

What bell did.
Journal of Phyalmonics A: Mathematical and Theoretical, 47(42):424010, 2014.

Tim WE Maudlin.
Completeness, supervenience and ontology.
Journal of Physics A: Mathematical and Theoretical, 40(12):3151, 2007b.

Brian McLaughlin and Karen Bennett.
Supervenience.
In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2018 edition, 2018.

Uwe Meixner.
The theory of ontic modalities, volume 13.
Walter de Gruyter, 2006.

David Hugh Mellor.
The matter of chance.
Cambridge University Press, 2004.

Christopher Menzel.
Actualism.
In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, summer 2018 edition, 2018.

David Miller.
Propensities and indeterminism.
Royal Institute of Philosophy Supplements, 39:121–147, 1995.

David Miller.
Critical rationalism: A restatement and defence.
Open Court, 2015.

Peter Milne.
Can there be a realist single-case interpretation of probability?
Erkenntnis, 25(2):129–132, 1986.

Thomas Müller, Antje Rumelnaperg, and Verena Wagner.
An introduction to real possibilities, indeterminism, and free will: three contingencies of the debate, 2018.

Wayne Myrvold.
Philosophical issues in quantum theory.
2016a.

Wayne C Myrvold.
Probabilities in statistical mechanics.
2016b.

Harold W Noonan.
Ej lowe on vague identity and quantum indeterminacy.
Analysis, 55(1):14–19, 1995.

Travis Norsen.
John s. bell's concept of local causality.
American Journal of Physics, 79(12):1261–1275, 2011.

John D Norton.
Causation as folk science.
2003.

John D Norton.
The dome: An unexpectedly simple failure of determinism.
Philosophy of Science, 75(5):786–798, 2008.

Timothy O'Connor and Christopher Franklin.
Free will.
In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2018 edition, 2018.

Samir Okasha.
Philosophical Theories of Probability.
The British Journal for the Philosophy of Science, 53(1):151–156, 03 2002.
ISSN 0007-0882.
doi: 10.1093/bjps/53.1.151.
URL `https://doi.org/10.1093/bjps/53.1.151`.

Graham Oppy.
Humean supervenience?
Philosophical studies, 101(1):77–105, 2000.

Flavia Padovani.
Coordination and measurement: What we get wrong about what reichenbach got right.
In *EPSA15 Selected Papers*, pages 49–60. Springer, 2017.

Michel Paty.
Reality and probability in mario bunge's treatise.
Studies on Mario Bunge's Treatise, pages 301–322, 1990.

Richard Pettigrew.
The dutch book arguments.
2020.

Itamar Pitowsky.
Quantum probability quantum logic.
1989.

Tomasz Placek and Leszek Wroński.
On infinite epr-like correlations.
Synthese, 167(1):1–32, 2009.

Henri Poincaré.
Science and method.
Courier Corporation, 2003.

Karl Popper.
A world of propensities.
1990.

Karl R Popper.
The propensity interpretation of probability.
The British journal for the philosophy of science, 10(37):25–42, 1959.

Willard V Quine.
On what there is.
The Review of Metaphysics, pages 21–38, 1948.

Frank P Ramsey.
Truth and probability.
In *Readings in Formal Epistemology*, pages 21–45. Springer, 1926.

Miklos Redei.
Kolmogorovian censorship hypothesis for general quantum probability theories.
Manuscrito-Revista Internacional de Filosofia, 33:365–380, 01 2010.

Hans Reichenbach.
The philosophy of space and time.
Courier Corporation, 1927.

Hans Reichenbach.
Philosophical foundations of probability.
In *Proceedings of the [First] Berkeley Symposium on Mathematical Statistics and Probability*.
The Regents of the University of California, 1949.

Hans Reichenbach.
The theory of relativity and a priori knowledge.
Univ of California Press, 1965.

Hans Reichenbach.
The theory of probability.
Univ of California Press, 1971.

Hans Reichenbach.
The direction of time, volume 65.
Univ of California Press, 1991.

Bertrand Russell.
On denoting.
Mind, 14(56):479–493, 1905.

Miklós Rédei.
Reichenbach's common cause principle and quantum correlations.
In *Non-locality and Modality*, pages 259–270. Springer, 2002.

Wesley C Salmon.
Propensities: A discussion review.
Erkenntnis, 14(2):183–216, 1979.

Wesley C Salmon.
Causality and explanation.
Oxford University Press, 1998.

Inaki San Pedro.
On free will and no-conspiracy.
In Tilman Sauer and Adrian Wüthrich, editors, *New Vistas on Old Problems: Recent Approaches to the Foundations of Quantum Mechanics*, pages 87–102. Max Planck Research Library for the History and Development of Knowledge, 2013.

Leonard J Savage.
The foundations of statistics.
Courier Corporation, 1972.

Jonathan Schaffer.
Deterministic chance?
The British journal for the philosophy of science, 58(2):113–140, 2007.

Erwin Schrödinger.

Discussion of probability relations between separated systems.
In *Mathematical Proceedings of the Cambridge Philosophical Society*, volume 31, pages 555–563. Cambridge University Press, 1935.

Wolfgang Schwarz.
Best system approaches to chance.
2014.

Wolfgang Schwarz.
No interpretation of probability.
Erkenntnis, 83(6):1195–1212, 2018.

Michiel P Seevinck.
Can quantum theory and special relativity peacefully coexist?
arXiv preprint arXiv:1010.3714, 2010.

Sydney Shoemaker.
Causal and metaphysical necessity.
Pacific Philosophical Quarterly, 79(1):59–77, 1998.

Jeff Speaks.
Theories of meaning.
Stanford Encyclopedia of Philosophy, 2010.

Peter Frederick Strawson.
Freedom and resentment and other essays.
Routledge, 2008.

Michael Strevens.
A closer look at the 'new'principle.
The British journal for the philosophy of science, 46(4):545–561, 1995.

Michael Strevens.
Objective probability as a guide to the world.
Philosophical Studies, 95(3):243–275, 1999.

Michael Strevens.
Bigger than chaos: Understanding complexity through probability.
Harvard University Press, 2009.

Michael Strevens.
Probability out of determinism.
Probabilities in physics, pages 339–364, 2011.

Mauricio Suárez.
Quantum selections, propensities and the problem of measurement.
The British Journal for the Philosophy of Science, 55(2):219–255, 2004.

Mauricio Suárez.
Quantum propensities.
Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics, 38(2):418–438, 2007.

Mauricio Suárez.
Propensities and pragmatism.
The Journal of Philosophy, 110(2):61–92, 2013.

Chris Swoyer.
The nature of natural laws.
Australasian Journal of Philosophy, 60(3):203–223, 1982.

László E Szabó.
Quantum mechanics in an entirely deterministic universe.
International Journal of Theoretical Physics, 34(8):1751–1766, 1995.

László E Szabó.
Critical reflections on quantum probability theory.
In *John von Neumann and the foundations of quantum physics*, pages 201–219. Springer, 2001a.

László E Szabó.
A physicalist interpretation of probability.
In *International Interdisciplinary Workshop on Determinism, Ringberg Castle (Rottach-Egern), Germany*, 2001b.

László E Szabó.
The einstein-podolsky-rosen argument and the bell inequalities.
arXiv preprint arXiv:0712.1318, 2007a.

László E Szabó.
Objective probability-like things with and without objective indeterminism.
Studies In History and Philosophy of Science Part B: Studies In History and Philosophy of Modern Physics, 38(3):626–634, 2007b.

László E Szabó.
What remains of probability?
In *The Present Situation in the Philosophy of Science*, pages 373–379. Springer, 2010.

Eran Tal.
Measurement in science.
In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2017 edition, 2017.

D Ter Haar.
Foundations of statistical mechanics.
Reviews of Modern Physics, 27(3):289, 1955.

Wolfgang Tittel, Jürgen Brendel, Hugo Zbinden, and Nicolas Gisin.
Violation of bell inequalities by photons more than 10 km apart.
Physical Review Letters, 81(17):3563, 1998.

Roderich Tumulka.
Comment on "the free will theorem".
Foundations of Physics, 37(2):186–197, 2007.

Roderich Tumulka.
Bohmian mechanics.
arXiv preprint arXiv:1704.08017, 2017.

Jos Uffink.
Can the maximum entropy principle be explained as a consistency requirement?
Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics, 26(3):223–261, 1995.

Lev Vaidman.
Quantum theory and determinism.
Quantum Studies: Mathematics and Foundations, 1(1-2):5–38, 2014.

Bas C Van Fraassen.
The charybdis of realism: Epistemological implications of bell's inequality.
Synthese, 52(1):25–38, 1982.

Bas C Van Fraassen.
Laws and symmetry.
1989.

Bas C Van Fraassen.
Scientific representation: Paradoxes of perspective, 2010.

Peter Van Inwagen.
An essay on free will.
1983.

Peter Van Inwagen.
Mcginn on existence.
The Philosophical Quarterly, 58(230):36–58, 2008.

John Venn.
The logic of chance: an essay on the foundations and province of the theory of probability, with especial reference to its logical bearings and its application to moral and social science, and to statistics.
Macmillan, 1888.

Barbara Vetter.
Potentiality: From dispositions to modality.
OUP Oxford, 2015.

Richard von Mises.
Probability, statistics, and truth.
Courier Corporation, 1964.

Richard Von Mises.
Mathematical theory of probability and statistics.
Academic Press, 2014.

David Wallace.
The emergent multiverse: Quantum theory according to the Everett interpretation.
Oxford University Press, 2012.

David Wallace.
Philosophy of quantum mechanics.
In *The Ashgate companion to contemporary philosophy of physics*, pages 22–104. Routledge, 2016.

David Wallace.
What is orthodox quantum mechanics?
In *Philosophers Look at Quantum Mechanics*, pages 285–312. Springer, 2019.

Brian Weatherson.
David lewis.
In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2016 edition, 2016.

Charlotte Werndl.
Determinism and indeterminism.
The Oxford handbook of philosophy of science, pages 210–232, 2016.

Takashi Yagisawa.
Possible objects.
In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, spring 2018 edition, 2018.

Anton Zeilinger.
Dance of the photons: from Einstein to quantum teleportation.
Farrar, Straus and Giroux, 2010.

Lyle Zynda.
Subjectivism.
In *The Oxford Handbook of Probability and Philosophy*. 2017.